

# **EFFECT OF SETBACK ON FUNDAMENTAL PERIOD OF RC FRAMED BUILDINGS**

*A THESIS*

*Submitted by*

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## **THESIS CERTIFICATE**

This is to certify that the thesis entitled “**EFFECT OF SETBACK ON FUNDAMENTAL PERIOD OF RC FRAMED BUILDINGS**” submitted by **VINAY MOHAN AGRAWAL** to the National Institute of Technology Rourkela for the award of the degree of Master of Technology is a bonafide record of research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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## ABSTRACT

**KEYWORDS:** *geometric irregularity, setback building, fundamental period, regularity index, correction factor.*

The motion of the ground during earthquake do not damage the building by impact or by any external force, rather it impacts the building by creating an internal inertial forces which is due to vibration of building mass. The magnitude of lateral force due to an earthquake depends mainly on inertial mass, ground acceleration and the dynamic characteristics of the building. To characterize the ground motion and structural behaviour, design codes provide a Response spectrum. Response spectrum conveniently describes the peak responses of structure as a function of natural vibration period. Therefore it is necessary to study of natural vibration period of building to understand the seismic response of building. The behaviour of a multi-storey framed building during strong earthquake motions depends on the distribution of mass, stiffness, and strength in both the horizontal and vertical planes of the building. In multi-storeyed framed buildings, damage from earthquake ground motion generally initiates at locations of structural weaknesses present in the lateral load resisting frames. In some cases, these weaknesses may be created by discontinuities in stiffness, strength or mass between adjacent storeys. Such discontinuities between storeys are often associated with sudden variations in the frame geometry along the height. There are many examples of failure of buildings in past earthquakes due to such vertical discontinuities. A common type of vertical geometrical irregularity in building structures arises from abrupt reduction of the lateral dimension of the building at specific levels of the elevation. This building category is known as the setback building. Setback buildings with geometric irregularity (both in

elevation and plan) are now increasingly encountered in modern urban construction. Setback buildings are characterised by staggered abrupt reductions in floor area along the height of the building, with consequent drops in mass, strength and stiffness. Height-wise changes in stiffness and mass render the dynamic characteristics of these buildings different from the ‘regular’ building. Many investigations have been performed to understand the behaviour of irregular structures as well as setback structures and to ascertain method of improving their performance.

This study presents the design code perspective of this building category. Almost all the major international design codes recommend dynamic analysis for design of setback buildings with scaled up base shear corresponding to the fundamental period as per the code specified empirical formula. However, the empirical equations of fundamental period given in these codes are a function of building height, which is ambiguous for a setback building. It has been seen from the analysis that the fundamental period of a setback building changes when the configuration of the building changes, even if the overall height remains the same. Based on modal analysis of 90 setback buildings with varying irregularity and height, the goal of this research is to investigate the accuracy of existing code-based equations for estimation of the fundamental period of setback buildings and provide suggestions to improve their accuracy.

This study shows that it is difficult to quantify the irregularity in a setback building with any single parameter. Also, this study indicates that there is very poor correlation between fundamental periods of three dimensional buildings with any of the parameters used to define

the setback irregularity by the previous researchers or design codes. The way design codes define setback irregularity by only geometry is found to be not adequate.

Period of setback buildings are found to be always less than that of similar regular building. Fundamental period of a framed building without infill stiffness depends not only on the height of the building but also on the bay width, irregularity and other structural and geometric parameters. It is not proper to relate the fundamental period of a framed building to height only as given in design code.

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# **CHAPTER 1**

## **INTRODUCTION**

### **1.1 BACKGROUND AND MOTIVATION**

The magnitude of lateral force due to an earthquake depends mainly on inertial mass, ground acceleration and the dynamic characteristics of the building. To characterize the ground motion and structural behaviour, design codes provide a Response spectrum. Response spectrum conveniently describes the peak responses of structure as a function of natural vibration period, damping ratio and type of founding soil. The determination of the fundamental period of structures is essential to earthquake design and assessment.

Seismic analysis of most structures is carried out using Linear Static (Equivalent Static) and Linear Dynamic (Response Spectrum) methods. Lateral forces calculated as per Equivalent Static Method depends on structural mass and fundamental period of structure. The empirical equations of the fundamental period of buildings given in the design codes are function of building height and base dimension of the buildings. Theoretically Response Spectrum Method uses modal analysis to calculate the natural periods of the building to compute the design base shear. However, some of the international codes (such as IS 1893:2002 and ASCE 7:2010) recommend to scale up the base shear (and other response quantities) corresponding to the fundamental period as per the code specified empirical formula, so as to improve this base shear (or any other response quantity) for Response Spectrum Analysis to make it equal to that of Equivalent

Static Analysis. Therefore, estimation of fundamental period using the code empirical formula is inevitable for seismic design of buildings.

Setback in buildings introduces staggered abrupt reductions in floor area along the height of the building. Figs 1.1 to 1.2 show typical examples of setback buildings. This building form is becoming increasingly popular in modern multi-storey building construction mainly because of its functional and aesthetic architecture. In particular, such a setback form provides for adequate daylight and ventilation for the lower storey in an urban locality with closely spaced tall buildings.



**Fig. 1.1:** Setback building: The Paramount Building at New York

This setback affects the mass, strength, stiffness, centre of mass and centre of stiffness of setback building. Dynamic characteristics of such buildings differ from the regular building due to changes in geometrical and structural property. Design codes are not clear about the definition of building height for computation of fundamental period. The bay-wise variation of height in setback building makes it difficult to compute natural period of such buildings.

With this background it is found essential to study the effect of setbacks on the fundamental period of buildings. Also, the performance of the empirical equation given in Indian Standard IS 1893:2002 for estimation of fundamental period of setback buildings is matter of concern for structural engineers. This is the primary motivation underlying the present study.



**Fig. 1.2:** Setback building: The Delhi secretariat building, India

## 1.2 OBJECTIVES

A detailed literature review is carried out to define the objectives of the thesis. This is discussed in detail in Chapter 2 and briefly summarised here. Design codes have not given particular attention to the setback building form. The research papers on setback buildings conclude that the displacement demand is dependent on the geometrical configuration of frame and concentrated in the neighbourhood of the setbacks for setback buildings. The higher modes significantly contribute to the response quantities of structure. There are a few literatures (Karavasilis *et. al.* 2008 and Sarkar *et. al.* 2010) on the definition and quantification of irregularity in setback buildings. This is an important parameter for estimation of fundamental period of setback buildings. There is a study (Sarkar *et. al.* 2010) on estimation of fundamental period of setback building frames. This study is limited only to plane frames and the formulation proposed in the study is difficult to be used for the actual three-dimensional setback buildings. Based on the literature review presented later, the salient objectives of the present study have been identified as follows:

- a) To perform a parametric study of the fundamental period of different types of reinforced concrete moment resisting frames (MRF) with varying number of stories, number of bays, configuration, and types of irregularity.
- b) To compare the fundamental periods of each structure calculated using code empirical equations and Rayleigh methods with fundamental period based on modal analysis.

### **1.3 SCOPE OF THE STUDY**

- a) The present study is limited to reinforced concrete (RC) multi-storeyed building frames with setbacks.
- b) Infill stiffness is not considered in the present study. However, associated mass and weight is assumed in the analysis.
- c) Setback buildings from 6 storeys to 30 storeys with different degrees of irregularity are considered.
- d) The buildings are assumed to have setback only in one direction.
- e) Soil-structure interaction effects are not considered in the present study. Column ends are assumed to be fixed at the foundation.

### **1.4 METHODOLOGY**

The steps undertaken in the present study to achieve the above-mentioned objectives are as follows:

- a) Carry out extensive literature review, to establish the objectives of the research work.
- b) Select an exhaustive set of setback building frame models with different heights (6 to 30 storeys), Bay width in both horizontal direction (5m, 6m and 7m bay width) and different irregularities (limit to 90 setback building models).
- c) Perform free vibration analysis for each of the 90 building models.
- d) Analysing the results of free vibration analysis



## **1.5 ORGANISATION OF THE THESIS**

This introductory chapter has presented the background, objective, scope and methodology of the present study. Chapter 2 starts with a description of the previous work done on setback moment-resisting frames by other researchers. Later in the chapter, a description of calculation of fundamental period and quantification of irregularity using different design codes are discussed. This chapter also discusses selected alternative methods reported in literature to overcome the existing limitations. Finally, a brief outline on the free vibration analysis is presented for better understanding of the results.

Chapter 3 describes the modelling aspects of setback buildings used in the present study for representing the actual behaviour of different structural components in the building frame. This Chapter then presents the different geometries of setback building considered in the study.

Chapter 4 begins with a presentation of general behaviour of setback buildings. It explains the variation of fundamental time period with the variation in setback geometry of the building. This chapter also explains the effect of bay width on fundamental period of the building. Finally, this chapter discusses on different code perspective on calculation of fundamental period and concludes some ambiguity related to code based empirical formulas.

Finally, Chapter 5 presents a summary including salient features, significant conclusions from this study and the future scope of research in this area.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

The literature review is conducted in two major areas. These are: (i) Response of setback buildings under seismic loading, effect of vertical irregularity on fundamental period of building and the quantification of setback and (ii) the recommendations proposed by seismic design codes on setback buildings. The first part of this chapter is devoted to a review of published literature related to response of irregular buildings under seismic loading. The response quantities include ductility demand, inter-story drift, lateral displacement, building frequencies and mode shapes. The second half of this chapter is devoted to a review of design code perspective on the estimation of fundamental period of setback building. This part describes different empirical formulas used in different design codes for the estimation of fundamental period, and the description and quantification of irregular buildings.

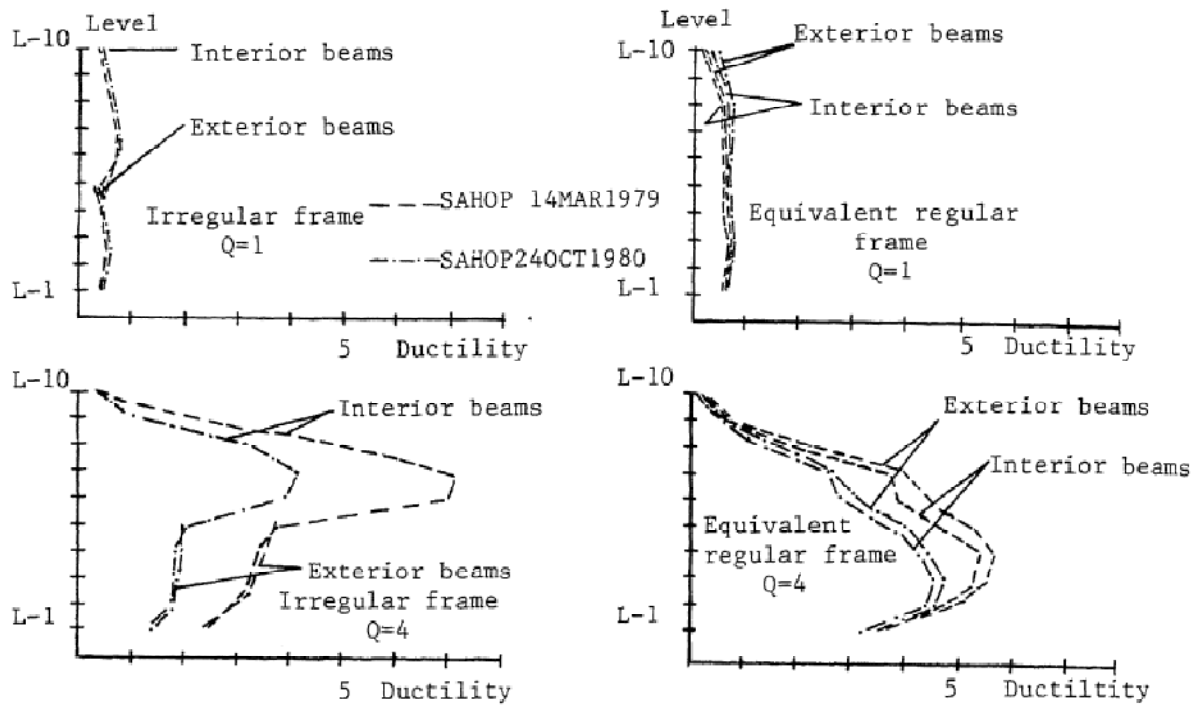
#### **2.2 RESEARCH ON SETBACK BUILDING**

The seismic response of vertically irregular building frames, which has been the subject of numerous research papers, started getting attention in the late 1970s. Vertical irregularities are characterized by vertical discontinuities in the geometry, distribution of mass, stiffness and strength. Setback buildings are a subset of vertically irregular buildings where there are discontinuities with respect to geometry. However, geometric irregularity also introduces discontinuity in the distribution of mass, stiffness and strength along the vertical direction.

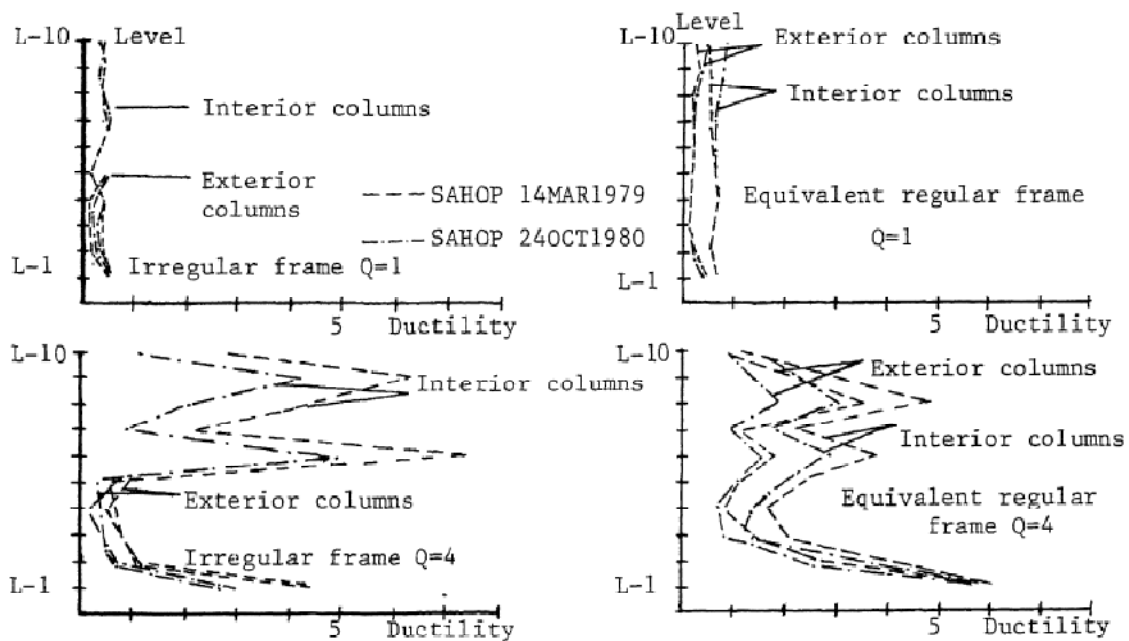
Majority of the studies on setback buildings have focused on the elastic response. Following is a brief review of the work that has been done on the seismic response of setback structures.

Humar *et. al.* (1977) studied the dynamic behaviour of multi-storey steel rigid-frame buildings with setback towers. The effects of setbacks upon the building frequencies and mode shapes were examined. Then the effects of setbacks on seismic response are investigated by analysing the response of a series of setback building frame models to the El Centro ground motion. Finally, the computed responses to the El Centro earthquake are compared with some code provisions dealing with the seismic design of setback buildings. The conclusions derived from the study include the following: The higher modes of vibration of a setback building can make a very substantial contribution to its total seismic response; this contribution increases with the slenderness of the tower. Some of the important response parameters for the tower portion of a setback building are substantially larger than for a related uniform building. For very slender towers, the transition region between the tower and the base may be subjected to very large storey shears.

Aranda *et. al.* (1984) studied the ductility demands of RC Frames irregular in height. The study focuses in inelastic behavior of RC Frames irregular in height when subjected to earthquake motion. For the numerical analysis static methods with different ductility factors were used. Two RC buildings of 30 m overall height was studied. One is the regular building with three bays of 5m each in both the horizontal direction. And the other one is irregular building with a tower of 5m bay width in both horizontal directions starting at mid height of the building and located centrally.



**Fig. 2.1:** Ductility demands in beams for the selected RC frames (Ref: Aranda,1984)



**Fig. 2.2:** Ductility demands in Columns for the selected RC frames (Ref: Aranda, 1984)

It was concluded that for both the models the ductility demand in the exterior beams are larger than in the interior beam as shown in Fig. 2.1 . Also the ductility demand is more in the interior columns of irregular frame as compared to the exterior ones as shown in Fig. 2.2. It is observed that ductility demands for setback structures is higher than that for the regular ones and this increase is more pronounced in the tower portions.

Shahrooz *et. al.* (1990) studied the effect of setbacks on the earthquake response of multistory buildings. The experimental and analytical study was undertaken. The test structure was six storeys, two bays by two bay reinforced concrete ductile moment resisting frames having 50 % setback at mid height. A modal analysis was performed using an acceleration spectrum. The variation of lateral displacements, inter storey drifts and lateral inertial forces in the direction parallel to setbacks. The displacement profile is found to be relatively smooth over the height without any irregularity in setback level, which is similar to a regular structure. They concluded the following points: The dynamic behavior of setback building is similar to that of a regular building with the only exception of torsion. Both the conventional dynamic and conventional static design methods prove to be inadequate to prevent concentration of damage in members near the setback. The definition of setback proposed in design codes is not appropriate. They proposed the design of setback buildings with increased strength in tower relative to base.

Wood *et. al.* (1992) investigated the seismic behavior of reinforced concrete frames with setbacks using the response of two small scale models. She studied the displacement, acceleration and shear response of setback frames during earthquake simulation. She found that the first mode dominates the displacement and shear response of setback buildings however the acceleration response is governed mainly by higher modes. She concluded that, the response of

setback structure is no different than that of the regular structure and hence it does not require different design considerations.

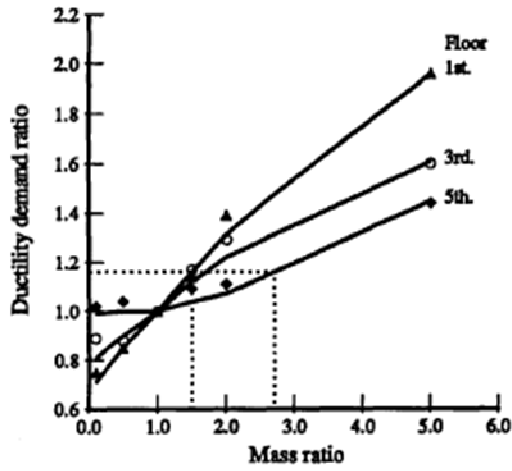
Wong *et. al.* (1994) studied the response of setback structures and they proposed a modification factor for adjusting the code period formulas so that it can more accurately determine the period of setback building. They also concluded that when the modal weight of a higher mode is larger than that of the fundamental mode, using the higher mode period for base shear calculation will result in unnecessarily conservative design.

Moehle *et. al.* (1986) carried out an experimental and analytical study for the response of strong base motions of reinforced concrete structures having irregular vertical configurations. For the purpose of study two small scales multistory reinforced concrete test structure was used. First test structure was named as FFW and it was a nine storey three bay frame with nine storey prismatic wall. The second test structure was identical to FFW except that the prismatic wall extended only up to the first floor level, this structure was named as FSW. Thus the test structures FFW and FSW represent the buildings having regular and irregular distributions of stiffness and strength in vertical planes. The response of these test structures were examined using four different analysis methods, those are: 1) Inelastic dynamic response history analysis, 2) Inelastic static analysis, 3) Elastic modal spectral analysis, and 4) Elastic static analysis. The study concluded that the dynamic analysis methods provides an indication of the maximum displacement response whereas the static method alone is not capable of indicating displacement amplitudes for a given seismic events. Inelastic analysis methods seem to be advantageous over elastic methods in recognizing the severity of the discontinuity in the structure with discontinues wall. They concluded that the main advantage of dynamic methods is that those are capable of estimating the maximum displacement response, whereas the static methods cannot be used for

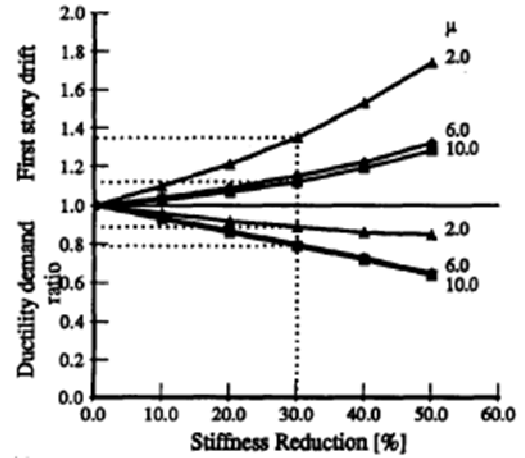
this purpose. Further, they inferred that the inelastic static and dynamic methods are superior to the elastic methods in interpreting the structural discontinuities.

Esteva *et. al.* (1992) studied the non linear dynamic response of building which has excess stiffness and strength at all other stories above the first one. The variables covered were: number of stories, fundamental period, form of the variation of story stiffness along height, ratio of post-yield to initial stiffness, in addition to the variable of primary interest, i.e., factor  $r$  expressing the ratio of the average value of the safety factor for lateral shear at the upper stories to that at the bottom story. The authors used shear-beam systems representative of buildings characterized by different number of stories and natural periods. The study included cases of stories with hysteretic bilinear behavior, both including and neglecting P-delta effects. He observed that the nature and magnitude of the influence of the ratio  $r$  on the maximum ductility demands at the first story depend on the low-strain fundamental period of the system. For very short periods those ductility demands may be reduced by about 30% when  $r$  grows from 1.0 to 3.0. For intermediate periods, ductility demands are little sensitive to  $r$ , but for longer periods those may reach the increments of 50 to 100% while  $r$  varies within the mentioned interval. It is also observed that the influence of  $r$  on the response of the first story is strongly enhanced if P-delta effects are taken into account.

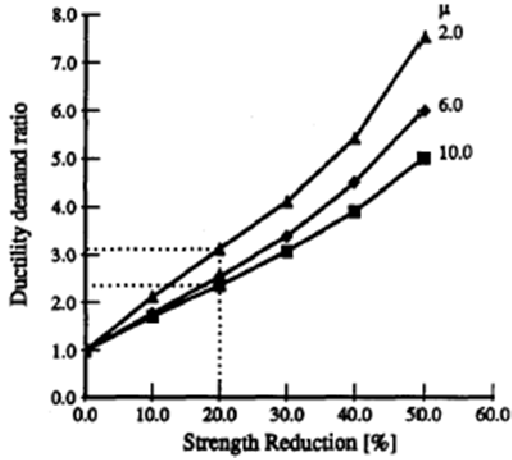
Valmundsson *et. al.* (1997) studied the two dimensional building frames with 5, 10 and 20 storey. They studied the earthquake response of these structures with non uniform mass, stiffness and strength distributions. Response from time history and equivalent lateral force methods are being compared. Based on this comparison they evaluated the requirements under which a structure can be considered regular and ELF procedure are applicable.



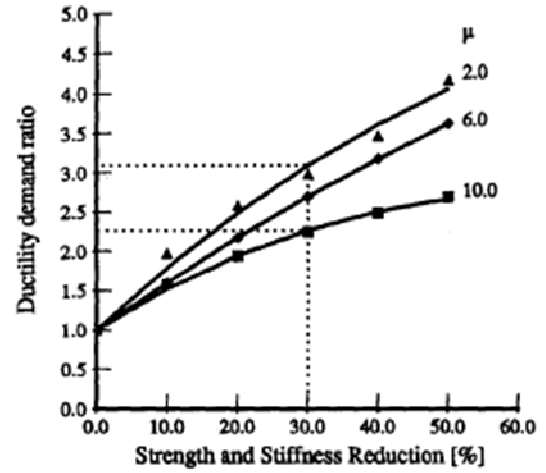
(a)



(b)



(c)



(d)

**Fig. 2.3:** (a) Maximum ductility demand for 5-story structure with mass irregularity and design ductility = 2; (b) Maximum ductility demand and first story drift for 20-story structure with stiffness irregularity; (c) Maximum ductility demand for 20-story structure with strength irregularity; (d) Maximum ductility demand for 20-story structure with strength and stiffness irregularities (Ref. Valmundsson, 1997)



They concluded that with the 50 % increase in the mass of one floor the ductility demand increases maximum by 20% as shown in Fig 2.3(a) depending on the design ductility. Reducing the stiffness of the first story by 30%, while keeping the strength constant, increases the first story drift by 20-40%, depending on the design ductility as shown in Fig 2.3(b). On reducing the strength of the first story by 20% the ductility demand increases by 100-200% as shown in Fig 2.3(c). Reducing the first story strength and stiffness proportionally by 30% increases the ductility demand by 80-200% as shown in Fig 2.3(d), depending on the design ductility. Thus strength criterion results in large increases in response quantities and is not consistent with the mass and stiffness requirements.

Al-Ali *et. al.* (1998) they studied the seismic response of buildings with vertical irregularity. They discussed on the quantification of effects of irregularity in mass, stiffness, strength and their combinations for seismic demands. Deformation demand i.e. roof drift and storey drift are also being studied. The analysis considered in the study is both elastic and inelastic dynamic analysis. Two dimensional, single bay, 10 story frame MDOF models designed according to strong beam weak column philosophy.

They found that seismic response due to mass irregularity is least, whereas the effect of strength irregularity is larger than the effect of stiffness irregularity. The seismic response was seen to be affected severely when the combined stiffness and strength irregularity is studied.

Chintanapakdee *et. al.* (2004) studied the seismic demands for vertically irregular and regular frames by non linear response history analysis. 48 irregular frames of 12 story height were designed and tested as per strong column weak beam philosophy. Three types of irregularities is considered for the study: Stiffness irregularity (KM), strength irregularity (SM), and combined

stiffness- and-strength irregularity (KS). The effect of vertical irregularity on storey drift and floor displacement were studied. They concluded the following points: The All the three types of irregularities KM, SM and KS influence the height-wise variation of story drifts, with the effects of strength irregularity being larger than stiffness irregularity, and the effects of combined-stiffness-and-strength irregularity being the largest among the three. Introducing a soft and/or weak story increases the story drift demands in the modified and neighboring stories and decreases the drift demands in other stories. On the other hand, a stiff and/or strong story decreases the drift demand in the modified and neighboring stories and increases the drift demands in other stories. Irregularity in upper stories has very little influence on the floor displacements. In contrast, irregularity in lower stories has significant influence on the height-wise distribution of floor displacements.

Sarkar *et. al.* (2010) proposes a new method of quantifying irregularity in stepped building frames, which accounts for dynamic characteristics i.e. mass and stiffness. This paper discusses some of the key issues regarding analysis and design of stepped buildings. They proposed a new approach for quantifying the irregularity in stepped building. It accounts for properties associated with mass and stiffness distribution in the frame. This approach is found to perform better than the existing measures to quantify the irregularity. Based on free vibration analysis of 78 stepped frames with varying irregularity and height, this study proposes a correction factor to the empirical code formula for fundamental period, to render it applicable for stepped buildings. They proposed a measure of vertical irregularity, called ‘regularity index’, accounting for the changes in mass and stiffness along the height of the building as a ratio of  $\Gamma_1$  and  $\Gamma_{ref}$ , Where,  $\Gamma_1$  is the 1<sup>st</sup> mode participation factor for the setback building frame under consideration and  $\Gamma_{ref}$  is

the 1<sup>st</sup> mode participation factor for the similar regular building frame without steps. The regularity index is mathematically expressed as follows:

$$\eta = \frac{\Gamma_1}{\Gamma_{1,ref}} \quad (2.1)$$

They concluded that regularity index increases with increase in number of storeys, and the rate of increase being stiffer when the number of storeys per step increases. Also, they concluded that, for any given number of storeys, the regularity index is least when the number of storeys per step is largest. Thus the proposed irregularity index is able to capture effectively the irregularity caused due to the various geometrical stepped configurations. In continuation of the study they propose to improve the code based empirical formula for estimating the fundamental period to render it applicable for stepped building. They defined a correction factor  $k$  for the empirical formula of IS 1893:2002 and modified it, as shown:

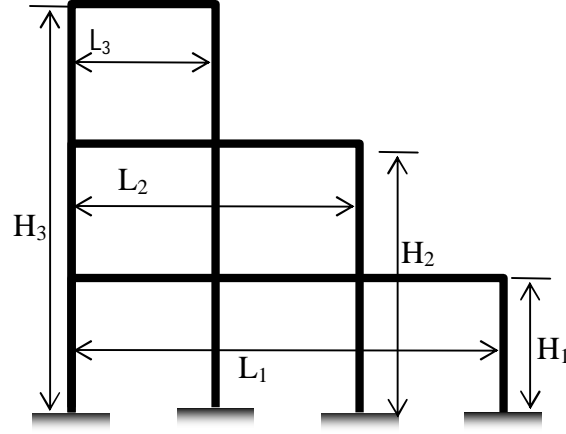
$$T = 0.075h^{0.75} \times k \quad (2.2)$$

$$k = \frac{T}{T_{ref}} = [1 - 2(1 - \eta)(2\eta - 1)] \quad \text{For } 0.6 \leq \eta \leq 1.0 \quad (2.3)$$

Where,  $h$  = overall building height in meter.

This index reported to be performing better than other measures. But this index is developed using two-dimensional building frames and not validated for actual buildings. Also, application of this index for three-dimensional building is difficult as 1<sup>st</sup> mode vibration for a setback building and 1<sup>st</sup> mode vibration for the similar regular building may not always be in same direction.

Karavasilis *et. al.* (2008) presented a study on the inelastic seismic response of plane steel moment resisting frames (MRF) with setbacks.



**Fig. 2.4:** Frame geometry for definition of irregularity indices (Karavasilis *et. al* 2008.)

The statistical analysis of the created response databank indicates that the number of stories, beam-to-column strength ratio, geometrical irregularity and limit state under consideration strongly influence the height wise distribution and amplitude of inelastic deformation demands. Nonlinear regression analysis is employed in order to derive simple formulae which reflect the aforementioned influences and offer, for a given strength reduction (or behaviour) factor, three important response quantities, i.e. the maximum roof displacement, the maximum inter storey drift ratio and the maximum rotation ductility along the height of the structure. They proposed an alternative approach to quantify the irregularity in a building frame due to the presence of steps. They defined two irregularity indices for stepped buildings,  $\Phi_s$ , and  $\Phi_b$  (for storey-wise and bay-wise stepping respectively) as follows:

$$\Phi_s = \frac{1}{n_s - 1} \sum_{i=1}^{n_s-1} \frac{L_i}{L_{i+1}} \quad (2.4)$$

$$\Phi_b = \frac{1}{n_b - 1} \sum_{i=1}^{n_b-1} \frac{H_i}{H_{i+1}} \quad (2.5)$$

where,  $n_s$  is the number of storeys of the frame and  $n_b$  is number of bays at the first storey of the frame.  $H_i$  and  $L_i$  are height and the width respectively of the  $i^{\text{th}}$  storey as shown in Fig 2.4.

These indices represent the irregularity in stepped frame in an improved manner compared to the code procedures. However, it is not convenient to use two indices to represent the irregularity of the same stepped building. Moreover, these indices are based on geometrical considerations alone. It was assumed that the column and beam sizes are uniform throughout their length and masses are uniformly distributed along the height and width of the frame. But this may not be the case in practical buildings. However unlike design code provisions, the irregularity indices prove to quantify the degree or amount of irregularity in a setback building.

Athanassiadou *et. al.* (2008) studied the seismic response of multistory reinforced concrete frame building irregular in elevation, but regular in plan. Irregular frames along with the similar regular frames were analyzed for the performance to both inelastic static pushover analysis and inelastic dynamic time history analysis for the same peak ground acceleration. To study the effect of design ductility the buildings were designed for high and medium ductility classes as per Euro code. The authors analyzed two dimensional ten storey plane frames and concluded that the response holds good only for medium-to-high rise building, irregular in elevation, but regular in plan. It is concluded that the effect of the ductility class on the cost of buildings is negligible, while the seismic performance of all irregular frames appears to be equally satisfactory. Buildings of both ductility classes seem to perform equally satisfactorily during the design earthquake. The over strength of the irregular frames is found to be similar to that of the regular

ones. This study does not recommend the use of inelastic static pushover analysis as it is unable to simulate higher mode effects on the structural response.

Young (2011) presented a study in the determination of the fundamental period of vibration of structures with geometric irregularities. This study investigated the fundamental periods of three different types of steel earthquake-resistant building structures: moment resisting frames (MRF), concentrically braced frames (CBF), and eccentrically braced frames (EBF) with varying geometric irregularities. A total of 24 MRFs, 12 CBFs, and 12 EBFs are designed and analyzed with ETABS v.9.7.2. The fundamental periods based on vibration theory for each example were compared with empirical equations, including design code equations as well as equations proposed in published literature. Based on the results obtained from vibration theory (Rayleigh equation), equations for the approximate fundamental periods are put forth for MRFs, CBFs, and EBFs which take into account vertical and horizontal irregularities.

They proposed two factors as  $\frac{H_{av}}{H}$  and  $\frac{D_{av}}{D}$ , and performed a regression along with overall building height to compute the fundamental period of the building. They defined  $\frac{H_{av}}{H}$  as the ratio of weighted average height of building to the overall height of the building and  $\frac{D_{av}}{D}$  as the ratio of weighted average width of building to the overall width of the building. They proposed an equation for the calculation of fundamental period the irregular steel MRF building of the form as shown below:

$$T = 0.071(H)^{0.75} \left( \frac{H_{av}}{H} \right)^{0.35} \left( \frac{D_{av}}{D} \right)^{0.20}$$

Through statistical comparison, it was found that a 3-variable power model which is able to account for irregularities resulted in a better fit to the Rayleigh data than equations which were dependent on height only. The proposed equations were validated through a comparison of available measured period data. For braced frames, the proposed equations were also compared with a database of examples from literature.

## **2.3 DESIGN CODE PERSPECTIVE**

Most of the available design codes for earthquake resistant building including IS 1893:2002, ASCE 7:2010, Euro code 8 or New Zealand code of practice, recommends an empirical formula for the determination of fundamental time period of building. Also the design codes define different types of irregular structures. The forthcoming sections discuss about the different approaches for calculating fundamental time period and the definition of irregularity as per available design codes.

### **2.3.1 Fundamental Time Period**

As per IS 1893:2002 buildings having simpler regular geometry and uniformly distributed mass and stiffness in plan as well as in elevation, suffer much less damage than buildings with irregular configurations. Design code recommends dynamic analysis to obtain the design seismic force for all irregular buildings. ASCE 7:2010 and Euro Code 8 specify similar guidelines. This chapter discusses about the analysis and design considerations of setback buildings only.

All design code recommends performing dynamic analysis on setback buildings to obtain design seismic forces, and its distribution to different levels along the height of the building. Codes recommend modifying the response quantities (such as base shear) to be scaled up to a factor if the response from dynamic analysis is less than the response calculated using the empirical equation of fundamental time period. The response quantity is to be scaled up by a factor which is the ratio of base shear using empirical equations to the base shear using dynamic analysis. Hence the use and adaptability of empirical equations of fundamental time period recommended in codes are discussed for irregular building frames in this study.

The fundamental time period of a building depends on building material, building type and overall dimension of the building. The fundamental period of building should be known to the designer before designing, so as to compute the design seismic base shear. The fundamental period for a building that is yet to be designed cannot be computed, so the design codes recommend certain empirical equations for determination of fundamental time period.

The fundamental natural period of vibration,  $T_a$  (in seconds), of a RC moment resisting frame of overall height  $h$  (in meter) without brick infill, as per IS 1893:2002 is given by:

$$T_a = 0.075h^{0.75} \quad (2.6)$$

For the determination of fundamental natural period of vibration,  $T$  (in seconds), of a RC moment resisting frame of overall height  $h_n$  (in meter) Uniform Building Code 94 recommends the formula as shown:

$$T = 0.0731(h_n)^{0.75} \quad (2.7)$$



In a similar way as per ASCE 7:2010, the approximate fundamental period  $T_a$  (in second) of a structure with over all height  $h_n$  (in meter) for a RC moment resisting frame building is given by:

$$T_a = 0.0466(h_n)^{0.9} \quad (2.8)$$

ASCE 7:2010 permits to determine fundamental period  $T_a$  (in second) of RC buildings from the following equation for structures not exceeding 12 stories in height provided storey height to be at least 3 m. The equation is of the following form where,  $N$  is the number of stories:

$$T_a = 0.1N \quad (2.9)$$

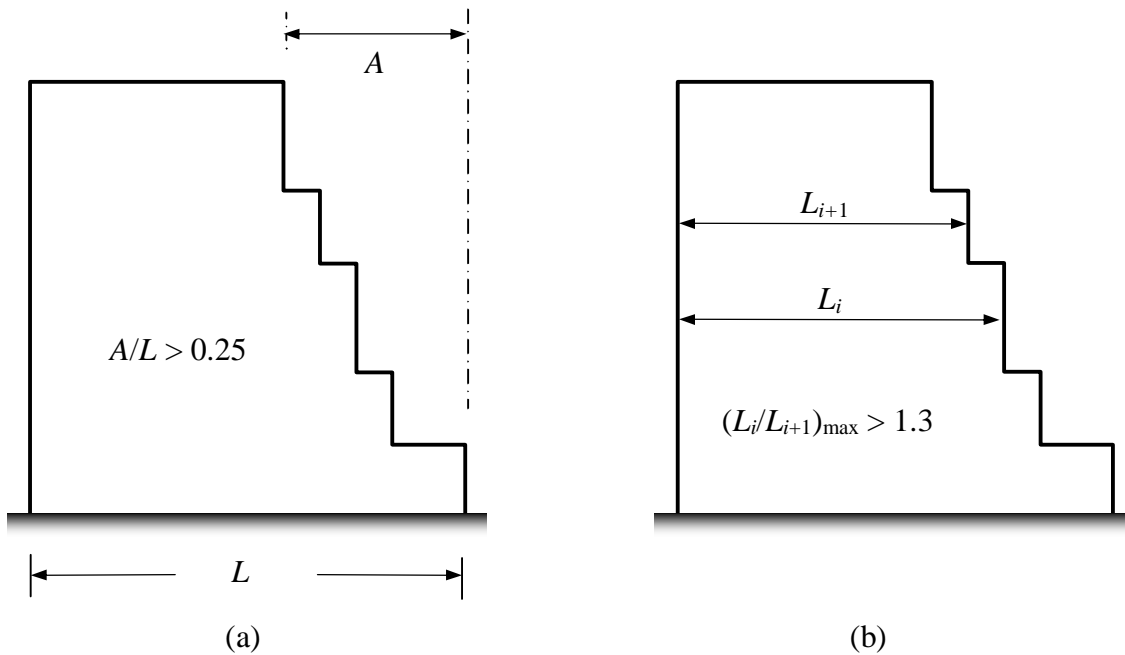
The code specifies that the fundamental period may be determined through an alternative substantiated analysis such as normal mode analysis or Rayleigh's method, both of which require the use of a computer program, making these theory-based methods of determining the fundamental period cumbersome for most practicing engineers. The Rayleigh equation is based on structural properties and deformational characteristics. Rayleigh's formula for the computation of fundamental period  $T$  (in second) is given by:

$$T = 2\pi \sqrt{\frac{\left( \sum_{i=1}^n w_i \delta_i^2 \right)}{\left( g \sum_{i=1}^n f_i \delta_i \right)}} \quad (2.10)$$

where  $w_i$  = the portion of the total seismic dead load located at or assigned to level  $i$ ;  $d_i$  = the horizontal displacement at level  $i$  relative to the base due to applied lateral forces;  $g$  = acceleration due to gravity; and  $f_i$  = the lateral force at level  $i$ . Rayleigh's formula is

recommended in New Zealand code of practice and, ASCE 7:2010 also recommends use of Rayleigh formula limiting the variation in results from Eq. 2.10 to be not more than 30 percent.

All the above empirical equation of fundamental period mentioned in codes (Eq. 2.6, Eq. 2.7 and Eq. 2.8) are function of overall building height and does not account for the stepped variations in height, applicable for setback buildings. However, Rayleigh's formula is based on the structural properties and deformational characteristics of the resisting elements and is a more rational approach. It is seen from analysis that the fundamental period of a stepped building changes when the nature of stepped configuration changes even although the height remains unchanged. Generally, the time period decreases with increased irregularity due to stepping. In many cases, this can lead to significant under-estimation of base shear particularly for tall buildings whose fundamental time period falls in the 'constant velocity' region of the response spectrum.



**Fig. 2.5:** Vertical geometric irregularity according to (a) IS 1893:2002 and (b) ASCE 7:2010 and BSSC 2003

### 2.3.2 Vertical Geometric Irregularity

All design codes defines plan irregularity and vertical irregularity as two major types of irregularity. Vertical geometric irregularity or Setback building is one among the vertical irregularity defined in all codes. As per IS 1893:2002, such building are to be considered as setback buildings where the horizontal direction of the lateral force resisting system in any story is more than 150 percent of that in its adjacent storey, as shown in Fig.2.5 (a). As per ASCE 7:2010, setback building is defined as, when the horizontal direction of the seismic force resisting system in any story is more than 130 percent of that in its adjacent storey, as shown in Fig.2.5(b). Design codes consider this ratio of lateral dimension of two adjacent stores as criteria to define vertical geometric irregularity. Design codes do not quantify the amount of irregularity in any setback building; it merely is a rule to distinguish regular and irregular building.

## 2.4 SUMMARY

This chapter is devoted to a review of published literature related to response of irregular buildings under seismic loading. Later this chapter is discusses design code perspective on the estimation of fundamental period of setback building. Empirical equations used in design codes, such as IS 1893:2002, ASCE 7:2010, Euro code 8 and Rayleigh method for the estimation of Fundamental period are discussed. The different code recommendations for the description and quantification of irregular buildings are discussed briefly.

The applicability of code based empirical formulas for calculation of fundamental period of setback buildings was no where discussed in the literature, except Sarkar *et. al.* (2010). This paper discussed the study on *plane frame* setback building as explained earlier. To further extend

the study this thesis discussed the effect of vertical irregularity on *three-dimensional* actual RC moment resisting frames.

## **CHAPTER 3**

### **STRUCTURAL MODELLING**

#### **3.1 INTRODUCTION**

The study in this thesis is based on analysis of a family of structural models representing vertically irregular multi-storeyed setback buildings. The first part of this chapter presents a summary of various parameters defining the computational models, the basic assumptions and the building geometries considered for this study. All the selected buildings were designed as per Indian Standards.

Later half of this chapter presents brief description of the design procedure followed in the present study. Free vibration analysis procedures of building system considered in the study also explained briefly at the end of the chapter.

#### **3.2 COMPUTATIONAL MODEL**

Modelling a building involves the modelling and assemblage of its various load-carrying elements. The model must ideally represent the mass distribution, strength, stiffness and deformability. Modelling of the material properties and structural elements used in the present study is discussed below.

##### **3.2.1 Material Properties**

M-20 grade of concrete and Fe-415 grade of reinforcing steel are used for all the frame models used in this study. Elastic material properties of these materials are taken as per

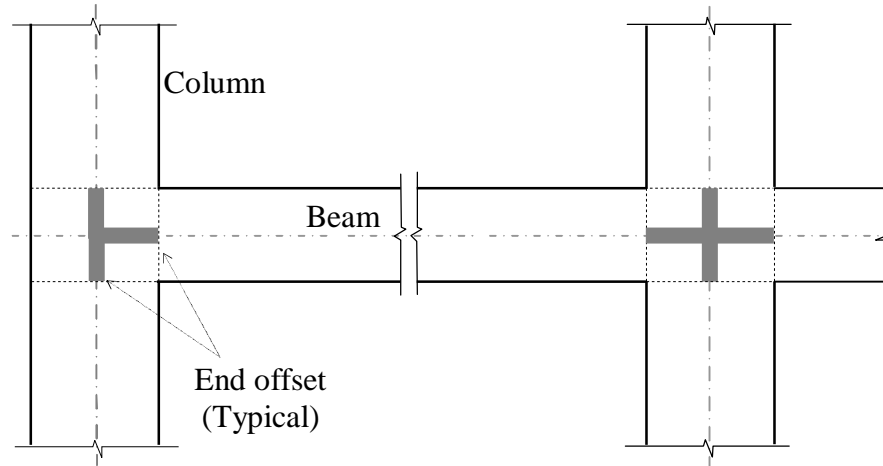
Indian Standard IS 456 (2000). The short-term modulus of elasticity ( $E_c$ ) of concrete is taken as:

$$E_c = 5000\sqrt{f_{ck}} \text{ MPa} \quad (3.1)$$

Where  $f_{ck} \equiv$  characteristic compressive strength of concrete cube in MPa at 28-day (20 MPa in this case). For the steel rebar, yield stress ( $f_y$ ) and modulus of elasticity ( $E_s$ ) is taken as per IS 456 (2000).

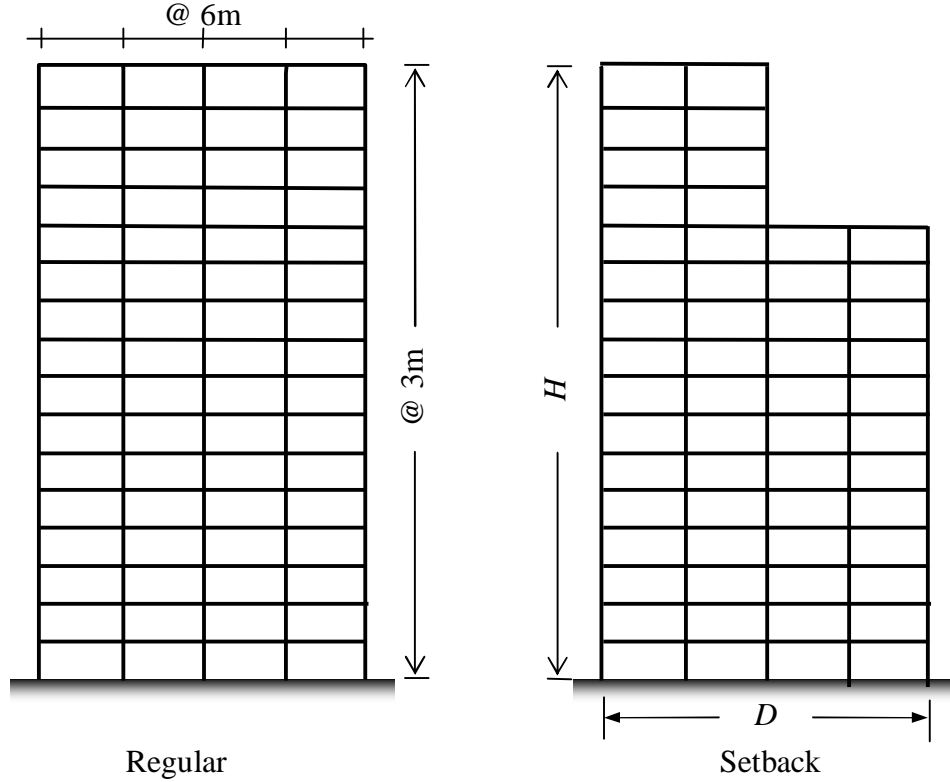
### 3.2.2 Structural Elements

Beams and columns are modelled by 2D frame elements. The beam-column joints are modelled by giving end-offsets to the frame elements, to obtain the bending moments and forces at the beam and column faces. The beam-column joints are assumed to be rigid (Fig. 3.1). The column end at foundation was considered as fixed for all the models in this study.



**Fig. 3.1:** Use of end offsets at beam-column joint

The structural effect of slabs due to their in-plane stiffness is taken into account by assigning ‘diaphragm’ action at each floor level. The mass/weight contribution of slab is modelled separately on the supporting beams.

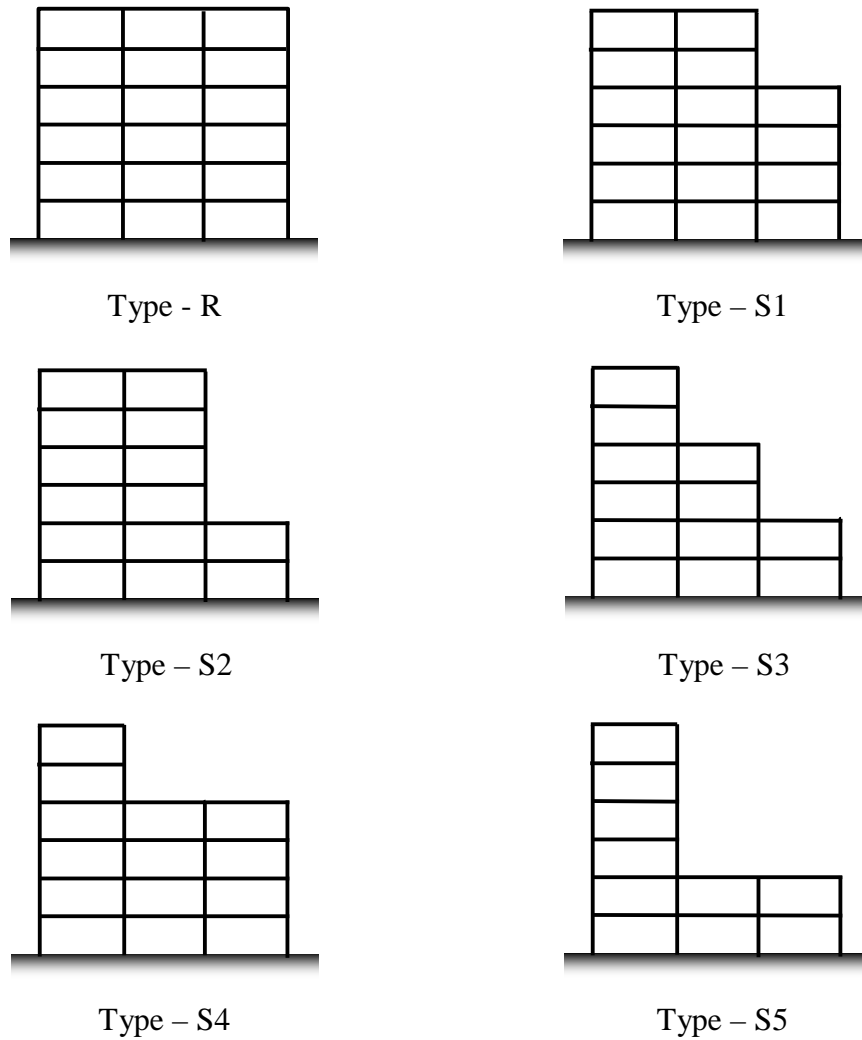


**Fig. 3.2:** Typical structural models used in the present study

### 3.3 BUILDING GEOMETRY

The study is based on three dimensional RC building with varying heights and widths. Different building geometries were taken for the study. These building geometries represent varying degree of irregularity or amount of setback. Three different bay widths, i.e. 5m, 6m and 7m (in both the horizontal direction) with a uniform three number of bays at base were considered for this study. It should be noted that bay width of 4m – 7m is the usual case, especially in Indian and European practice. Similarly, five different height categories were considered for the study, ranging from 6 to 30 storeys,

with a uniform storey height of 3m. Altogether 90 building frames with different amount of setback irregularities due to the reduction in width and height were selected. The building geometries considered in the present study are taken from literature (Karavasisis *et. al.*, 2008). The regular frame, without any setback, is also studied shown in Fig. 3.2.



**Fig 3.3:** Typical building elevations for six-storey building variants (R, S1 to S5)



There are altogether six different building geometries, one regular and five irregular, for each height category are considered in the present study. Fig 3.3 presents the elevation of all six different geometries of a typical six storey building. The buildings are three dimensional, with the irregularity in the direction of setback, in the other horizontal direction the building is just repeating its geometric configuration. Setback frames are named as S1, S2, S3, S4 and S5 depending on the percentage reduction of floor area and height as shown in the Fig. 3.3. The regular frame is named as R. The exact nomenclature of the buildings considered are expressed in the form of S-X-Y, where S represents the type of irregularity (*i.e.*, S1 to S5 or R). X represents the number of storeys and Y represents the bay width in both the horizontal direction. For example S3-18-6 represents the building with S3 type of irregularity, having 18 numbers of stories and bay width of 6m in both the horizontal direction. For all the other setback buildings the reduction in height and reduction of width will be consistent with reductions as explained in Fig.3.3. For example a S3-18-5 will have plan dimension of 3 bay by 3 bay at the base and will continue up to 6<sup>th</sup> floor. Plan dimension will reduce to 2 bay by 2 bay from 7<sup>th</sup> to 12<sup>th</sup> floor, and it will further reduce to 1 bay by 1 bay from 13<sup>th</sup> floor to 18<sup>th</sup> floor. The setbacks are considered in one horizontal direction only; the building is made three dimensional by repeating these bays in other horizontal direction. The frames are designed with M-20 grade of concrete and Fe-415 grade of reinforcing steel as per prevailing Indian Standards. Gravity (dead and imposed) load and seismic load corresponding to seismic zone II of IS 1893:2002 are considered for the design. The cross sectional dimensions of beams and columns are taken as shown in Table 3.1. The slab thickness is considered to be 120mm for all the buildings, Infill walls in the exterior

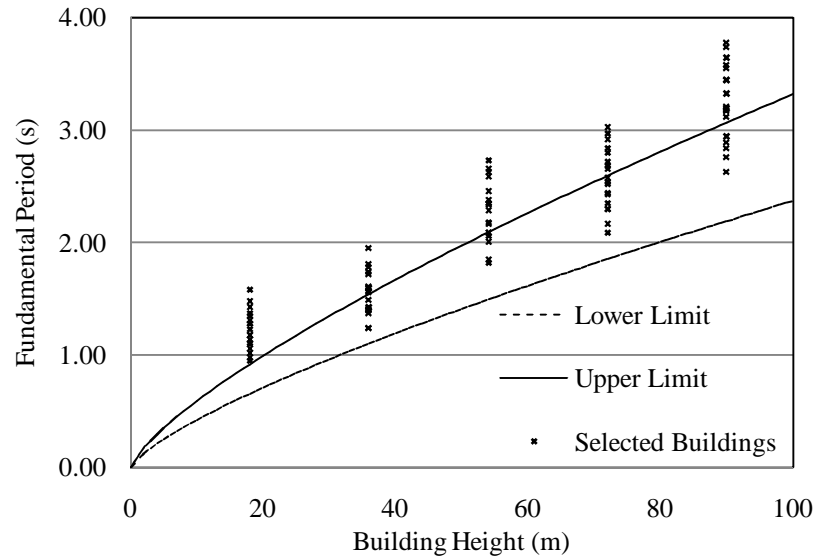
faces of all the buildings are assumed as of 230mm thickness and of 120mm thickness for all the inner infill walls. The parapet wall is assumed to be of 230 mm thickness and of 1000mm height for all the selected buildings.

**Table 3.1:** Dimensions of beams and columns for different buildings

Building Type according to number to stories	Column dimension	Beam dimension
Six-storey building	400 mm $\times$ 400 mm	300 mm $\times$ 450 mm
Twelve-storey building	600 mm $\times$ 600 mm	450 mm $\times$ 600 mm
Eighteen storey building	800 mm $\times$ 800 mm	450 mm $\times$ 600 mm
Twenty four-storey building	1000 mm $\times$ 1000 mm	450 mm $\times$ 750 mm
Thirty-storey building	1200 mm $\times$ 1200 mm	600 mm $\times$ 750 mm

The structures are modelled by using computer software SAP-2000 (v12) as explained in Section 3.2. Modal analyses were performed to check if the selected frames represent realistic building models. It is found that the selected buildings cover a wide fundamental period range of 0.95s – 3.78s. It may be noted that the fundamental period versus overall height variation of all the selected frames are consistent with the empirical relationships presented by Goel and Chopra (1997) as shown in Fig. 3.4. This shows that the models

selected for this study can be interpreted as being representative of general moment resisting RC frame behaviour for six to thirty-storey buildings, as established by Goel and Chopra (1997).



**Fig. 3.4:** Fundamental period versus overall height variation of all the selected frames

### 3.4 LINEAR DYNAMIC ANALYSIS

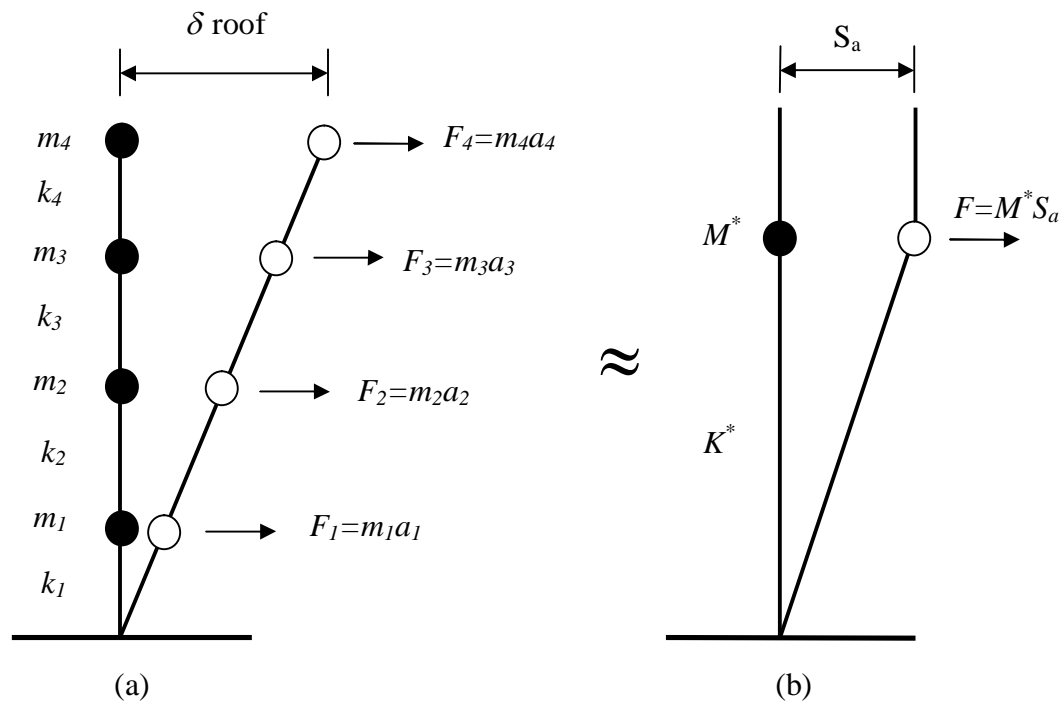
Symmetrical buildings with uniform mass and stiffness distribution behave in a fairly predictable manner, whereas buildings that are asymmetrical or with areas of discontinuity or irregularity do not. For such buildings, dynamic analysis is used to determine significant response characteristics such as (1) the effect of the structure's dynamic characteristics on the vertical distribution of lateral forces; (2) the increase in dynamic loads due to Torsional motions; and (3) the influence of higher modes, resulting in an increase in story shears and deformations.

Static method specified in building codes are based on single-mode response with simple corrections for including higher mode effects. While appropriate for simple regular structures, the simplified procedures do not take into account the full range of seismic behaviour of complex structures. Therefore, dynamic analysis is the preferred method for the design of buildings with unusual or irregular geometry.

Two methods of dynamic analyses are permitted: (1) elastic response spectrum analysis and (2) elastic or inelastic time history analysis. The response spectrum analysis is the preferred method because it is easier to use. The time history procedure is used if it is important to represent inelastic response characteristics or to incorporate time dependent effects when computing the structure's dynamic response.

Buildings are analysed as multi degree of freedom (MDOF) systems by lumping storey masses at intervals along the length of a vertically cantilevered pole. During vibration, each mass will deflect in one direction or another. For higher modes of vibration, some masses may move in opposite direction. Or all masses may simultaneously deflect in the same direction as in the fundamental mode. An idealized MDOF system has a number of modes equal to the number of masses. Each mode has its own natural period of vibration with a unique mode shaped by a line connecting the deflected masses. When ground motion is applied to the base of the multi mass system, the deflected shape of the system is a combination of all mode shapes, but modes having periods near predominant periods of the base motion will be excited more than the other modes. Each mode of the multi mass system can be represented by an equivalent single mass system having generalised values  $M$  and  $K$  for mass and stiffness, respectively. The generalised values represent the equivalent combined effects of story masses  $m_1, m_2, \dots$  and  $k_1, k_2, \dots$ . This concept, shown

in Fig. 3.5, provides a computational basis for using response spectra based on single mass systems for analysing multi-storeyed building, we can use the response spectra of a single degree of freedom (SDOF) system for computing the deflected shape, story acceleration, forces, and overturning moments. Each predominant mode is analysed separately and the results are combined statically to compute the multimode response.



**Fig. 3.5:** Representation of a multi-mass system by a single mass system: (a) fundamental mode of a multi mass system and (b) equivalent single mass system.

Buildings with symmetrical shape, stiffness, and mass distribution and with vertical continuity and uniformity behave in a fairly predictable manner, whereas when buildings are eccentric or have areas of discontinuity or irregularity; the behavioural characteristics are very complex. The predominant response of the building may be skewed from the

apparent principal axes of the building. The resulting torsional response as well as the coupling or interaction of the two translational directions of response must be considered by using a 3D model for the analysis.

For a building that is regular and essentially symmetrical, a 2D model is generally sufficient. When the floor plan aspect ratio (length to width) of the building is large, torsion response may be predominant, thus requiring a 3D analysis in an otherwise symmetrical and regular building. For most building, inelastic response can be expected to occur during a major earthquake, implying that an inelastic analysis is more proper for design. However, in spite of the availability of non linear elastic programs, they are not used in typical design practice because (1) their proper use requires the knowledge of their inner workings and theories, (2) the results produced are difficult to interpret and apply to traditional design criteria, and (3) the necessary computations are expensive. Therefore, analyses in practice typically use linear elastic procedures based on the response spectrum method (Taranath, 2010).

### **3.4.1 Modal Analysis**

When free vibration is under consideration, the structure is not subjected to any external excitation (force or support motion) and its motion is governed only by the initial conditions. There are occasionally circumstances for which it is necessary to determine the motion of the structure under conditions of free vibration. However, the analysis of the structure in free motion provides the most important dynamic properties of the structure which are the natural frequencies and the corresponding modal shapes.

By considering the fact that the damping levels are usually very small in structural systems, the equation of free vibration can be written as:

$$M\ddot{v} + Kv = 0 \quad (3.2)$$

Looking for a solution in the form of  $v_i = q(t)\phi_i, i = 1, 2, \dots, N$ , where the dependence on time and that on space variables can be separated. Substituting for  $v$ , the equation of motion changes to the following form:

$$M\{\phi\}\ddot{q}(t) + K\{\phi\}q(t) = 0 \quad (3.3)$$

This is a set of  $N$  simultaneous equations of the type

$$\sum_{j=1}^N m_{ij}\phi_j\ddot{q}(t) + \sum_{j=1}^N k_{ij}\phi_jq(t) = 0; i = 1, 2, \dots, N \quad (3.4)$$

Where the separation of variables leads to:

$$-\frac{\ddot{q}(t)}{q(t)} = \frac{\sum_{j=1}^N k_{ij}\phi_j}{\sum_{j=1}^N m_{ij}\phi_j}; i = 1, 2, \dots, N \quad (3.5)$$

As the terms on either side of this equation is independent of each other, this quantity can hold good only when each of these terms are equal to a positive constant, say  $\omega^2$ . Thus we have,

$$\ddot{q}(t) + \omega^2 q(t) = 0 \quad (3.6)$$

$$\sum_{j=1}^N (k_{ij} - \omega^2 m_{ij}) \phi_j = 0; i = 1, 2, \dots, N \quad (3.7)$$

The solution of Eq. 3.6 is  $q(t) = \sin(\omega t - \alpha)$  a harmonic of frequency  $\omega$ . Hence the motion of all coordinates is harmonic with same frequency and same phase difference  $\alpha$ . The above equation is a set of  $N$  simultaneous linear homogenous equations in unknowns of  $\phi_j$ . The problem of determining constant  $(\omega^2)$  for which the Eq. 3.7 has a non-trivial solution is known as the *characteristic value* or *Eigen value* problem. The Eigen value problem may be rewritten, in matrix notations as,

$$(K - \omega^2 M)\{\phi\} = 0 \quad (3.8)$$

A non-trivial solution for the Eq. 3.8 is feasible when only the determinant of the coefficient matrix vanishes, *i.e.*,

$$|K - \omega^2 M| = 0 \quad (3.9)$$

The expansion of the determinant in Eq. 3.9 yields an algebraic equation of  $N^{\text{th}}$  order in  $\omega^2$ , which is known as the characteristic equation. The roots of characteristic equation are known as the Eigen values and the positive square roots of these Eigen values are known as the *natural frequencies*  $(\omega_i)$  of the MDOF system. It is only at these  $N$  frequencies that the system admits synchronous motion at all coordinates. For stable structural systems with symmetric and positive stiffness and mass matrices the Eigen values will always be real and positive. For each Eigen values the resulting synchronous motion has a distinct shape and is known as *natural/normal mode shape* or *eigenvector*. The normal modes are as much a characteristic of the system as the Eigen values are.



They depend on the inertia and stiffness, as reflected by the coefficients  $m_{ij}$  and  $k_{ij}$ . These shapes correspond to those structural configurations, in which the inertia forces imposed on the structure due to synchronous harmonic vibrations are exactly balanced by the elastic restoring forces within the structural system. These eigenvectors are determined as the non-trivial solution of Eq. 3.8.

### 3.4.2 Mode Participation Factor

The forced vibration of MDOF system excited by support motion is described by the coupled system of differential equation as:

$$M\ddot{v} + C\dot{v} + Kv = -Mr\ddot{v}_g \quad (3.10)$$

Where  $\ddot{v}_g$  denotes ground acceleration,  $v$  is the vector of structural displacements *relative* to the ground displacements, and  $r$  is a vector of influence coefficients. The  $i^{th}$  element of vector  $r$  represents the displacement of  $i^{th}$  degree of freedom due to a unit displacement of the base. The nature of this equation is similar to that of standard forced vibration problem. Hence this can be solved using mode-superposition method and the equation can be decoupled as:

$$\ddot{q}_r + 2\zeta_r\omega_r\dot{q}_r + \omega_r^2q_r = -\Gamma_r\ddot{v}_g, \forall r = 1, 2, \dots, N \quad (3.11)$$

Where,  $\Gamma_r = \frac{\{\phi^{(r)}\}^T Mr}{\{\phi^{(r)}\}^T M \{\phi^{(r)}\}}$  is known as the *mode – participation Factor* for the  $r^{th}$  mode.

### **3.5 SUMMARY**

This chapter presents details of the structural models of selected RC framed buildings. It also describes the selected building geometries used in the present study. The selected buildings are representing the realistic three dimensional buildings of 6-30 storeys. Free vibration analysis method used in the present study is also explained in this chapter.

## **CHAPTER 4**

### **RESULTS AND DISCUSSIONS**

#### **4.1 INTRODUCTION**

All the selected building models with different setback irregularities are analyzed for linear dynamic behaviour using commercial software SAP2000 (v12). This chapter presents the analysis results and relevant discussions. According to the objectives of the present study, the results presented here are focussed on fundamental time period of selected setback buildings. The details of the selected buildings and the outline of the analysis procedure followed in this study are outlined in Chapter 3.

##### **4.2.1 FUNDAMENTAL TIME PERIOD FOR SETBACK BUILDINGS**

The fundamental time periods of all the 90 selected setback buildings were calculated using different methods available in literature including code based empirical formulas. These methods are explained in Chapter 2. Fundamental period of these buildings were also calculated using modal analysis. Modal analysis procedure is explained in Chapter 3.

The fundamental periods for all the selected setback buildings as obtained from different methods available in literature are tabulated in Tables 4.1 - 4.3. Table 4.1 presents the results of buildings with 5m bay width, Table 4.2 presents the results of buildings with 6m bay width whereas the Table 4.3 presents the results of buildings with 7m bay width. The fundamental periods presented here are computed as per different code empirical equations such as IS 1893:2002 (Eq. 2.6), UBC 94 (Eq. 2.7), ASCE 7 (Eqs. 2.8 and 2.9) as well as Rayleigh Method (Eq. 2.10), and period obtained from modal analysis.

**Table 4.1:** Fundamental period (s) of setback buildings with 5 m bay width

Building Designation	Height (m)	$T_{IS1893}$ (Eq. 2.6)	$T_{UBC.94}$ (Eq. 2.7)	$T_{ASCE.7}$ (Eq. 2.8)	$T_{ASCE.7}$ (Eq. 2.9)	$T_{Rayleigh}$ (Eq. 2.10)	$T_{Modal}$
R-6-5	18	0.66	0.64	0.63	0.60	1.1	1.17
S1-6-5	18	0.66	0.64	0.63	0.60	1.02	1.05
S2-6-5	18	0.66	0.64	0.63	0.60	1.02	1.09
S3-6-5	18	0.66	0.64	0.63	0.60	0.9	0.95
S4-6-5	18	0.66	0.64	0.63	0.60	0.93	0.97
S5-6-5	18	0.66	0.64	0.63	0.60	0.94	1.01
R-12-5	36	1.10	1.07	1.17	1.20	1.32	1.49
S1-12-5	36	1.10	1.07	1.17	1.20	1.21	1.37
S2-12-5	36	1.10	1.07	1.17	1.20	1.29	1.4
S3-12-5	36	1.10	1.07	1.17	1.20	1.09	1.24
S4-12-5	36	1.10	1.07	1.17	1.20	1.11	1.24
S5-12-5	36	1.10	1.07	1.17	1.20	1.21	1.40
R-18-5	54	1.49	1.46	1.69	1.80	1.89	2.18
S1-18-5	54	1.49	1.46	1.69	1.80	1.73	2.00
S2-18-5	54	1.49	1.46	1.69	1.80	1.86	2.08
S3-18-5	54	1.49	1.46	1.69	1.80	1.73	1.84
S4-18-5	54	1.49	1.46	1.69	1.80	1.70	1.82
S5-18-5	54	1.49	1.46	1.69	1.80	1.95	2.16
R-24-5	72	1.85	1.81	2.19	2.40	2.04	2.44
S1-24-5	72	1.85	1.81	2.19	2.40	1.98	2.29
S2-24-5	72	1.85	1.81	2.19	2.40	2.10	2.43
S3-24-5	72	1.85	1.81	2.19	2.40	1.95	2.16
S4-24-5	72	1.85	1.81	2.19	2.40	1.89	2.09
S5-24-5	72	1.85	1.81	2.19	2.40	2.19	2.72
R-30-5	90	2.19	2.14	2.67	3.00	2.57	3.18
S1-30-5	90	2.19	2.14	2.67	3.00	2.34	2.89
S2-30-5	90	2.19	2.14	2.67	3.00	2.51	3.12
S3-30-5	90	2.19	2.14	2.67	3.00	2.20	2.76
S4-30-5	90	2.19	2.14	2.67	3.00	2.12	2.63
S5-30-5	90	2.19	2.14	2.67	3.00	2.8	3.55

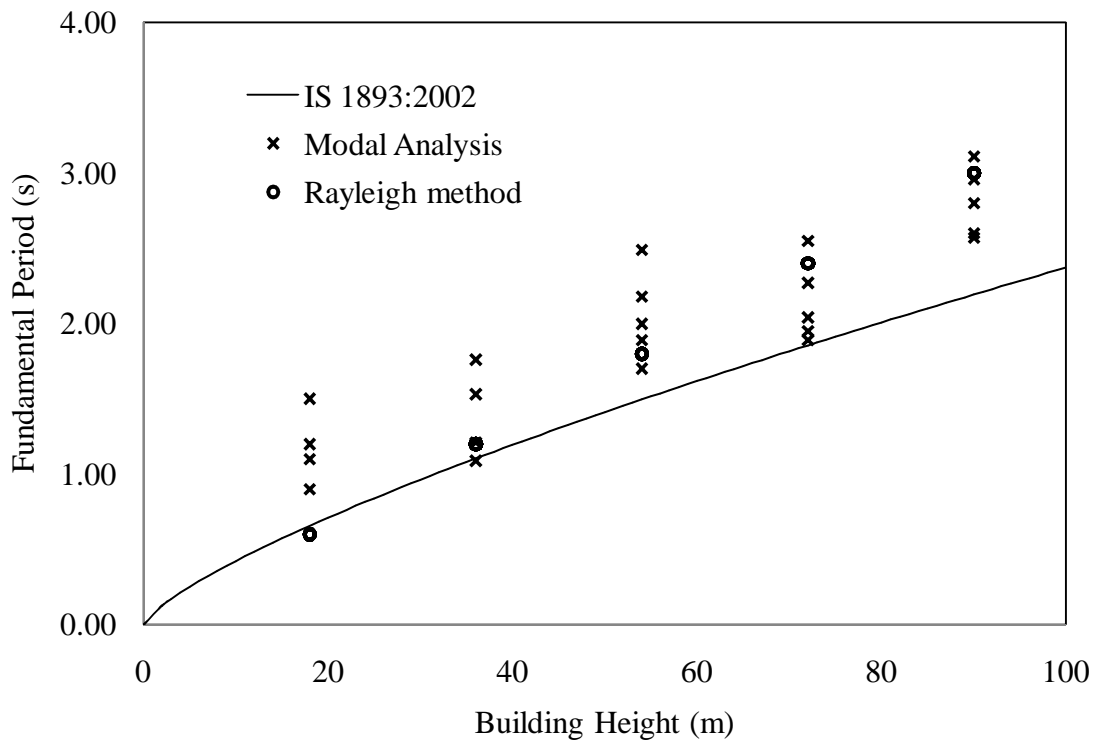
**Table 4.2:** Fundamental period (s) of setback buildings with 6 m bay width

Building Designation	Height (m)	$T_{IS1893}$ (Eq. 2.6)	$T_{UBC.94}$ (Eq. 2.7)	$T_{ASCE.7}$ (Eq. 2.8)	$T_{ASCE.7}$ (Eq. 2.9)	$T_{Rayleigh}$ (Eq. 2.10)	$T_{Modal}$
R-6-6	18	0.66	0.64	0.63	0.60	1.30	1.37
S1-6-6	18	0.66	0.64	0.63	0.60	1.20	1.23
S2-6-6	18	0.66	0.64	0.63	0.60	1.19	1.28
S3-6-6	18	0.66	0.64	0.63	0.60	1.06	1.11
S4-6-6	18	0.66	0.64	0.63	0.60	1.09	1.13
S5-6-6	18	0.66	0.64	0.63	0.60	1.09	1.17
R-12-6	36	1.10	1.07	1.17	1.20	1.53	1.72
S1-12-6	36	1.10	1.07	1.17	1.20	1.4	1.57
S2-12-6	36	1.10	1.07	1.17	1.20	1.42	1.60
S3-12-6	36	1.10	1.07	1.17	1.20	1.25	1.41
S4-12-6	36	1.10	1.07	1.17	1.20	1.28	1.42
S5-12-6	36	1.10	1.07	1.17	1.20	1.36	1.56
R-18-6	54	1.49	1.46	1.69	1.80	2.18	2.45
S1-18-6	54	1.49	1.46	1.69	1.80	2.00	2.28
S2-18-6	54	1.49	1.46	1.69	1.80	2.05	2.35
S3-18-6	54	1.49	1.46	1.69	1.80	1.80	2.08
S4-18-6	54	1.49	1.46	1.69	1.80	1.81	2.06
S5-18-6	54	1.49	1.46	1.69	1.80	2.02	2.37
R-24-6	72	1.85	1.81	2.19	2.40	2.27	2.68
S1-24-6	72	1.85	1.81	2.19	2.40	2.15	2.52
S2-24-6	72	1.85	1.81	2.19	2.40	2.23	2.65
S3-24-6	72	1.85	1.81	2.19	2.40	1.97	2.35
S4-24-6	72	1.85	1.81	2.19	2.40	2.13	2.30
S5-24-6	72	1.85	1.81	2.19	2.40	2.25	2.84
R-30-6	90	2.19	2.14	2.67	3.00	2.82	3.45
S1-30-6	90	2.19	2.14	2.67	3.00	2.57	3.19
S2-30-6	90	2.19	2.14	2.67	3.00	2.71	3.32
S3-30-6	90	2.19	2.14	2.67	3.00	2.37	2.94
S4-30-6	90	2.19	2.14	2.67	3.00	2.35	2.84
S5-30-6	90	2.19	2.14	2.67	3.00	2.80	3.64

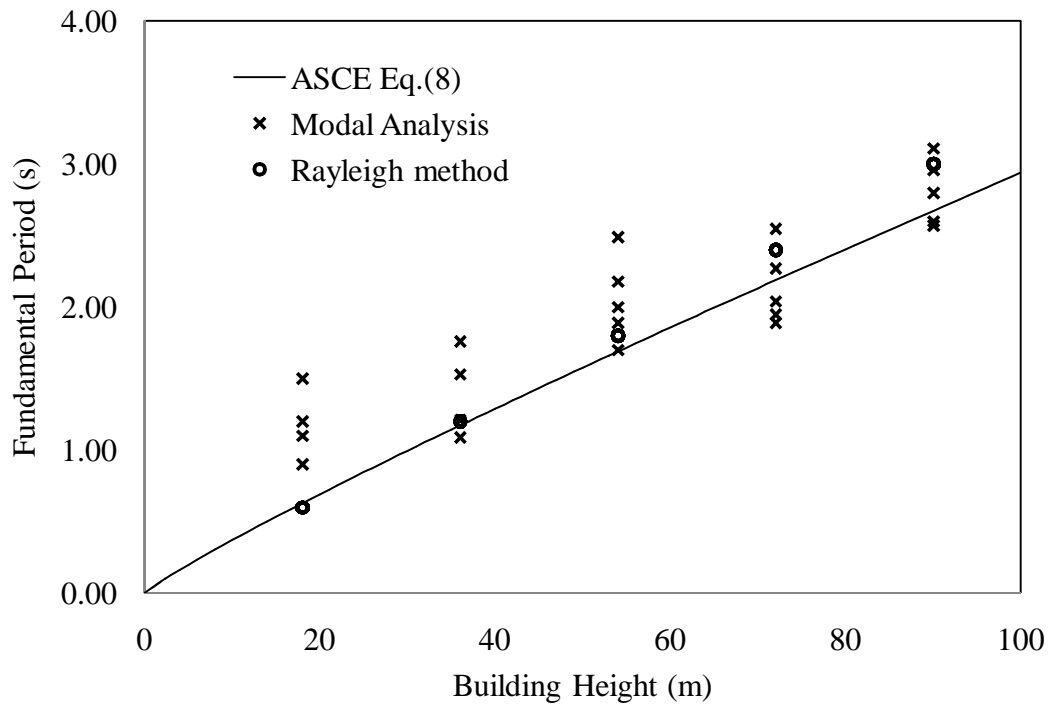
**Table 4.3:** Fundamental period (s) of setback buildings with 7 m bay width

Building Designation	Height (m)	$T_{IS1893}$ (Eq. 2.6)	$T_{UBC.94}$ (Eq. 2.7)	$T_{ASCE.7}$ (Eq. 2.8)	$T_{ASCE.7}$ (Eq. 2.9)	$T_{Rayleigh}$ (Eq. 2.10)	$T_{Modal}$
R-6-7	18	0.66	0.64	0.63	0.60	1.50	1.58
S1-6-7	18	0.66	0.64	0.63	0.60	1.35	1.42
S2-6-7	18	0.66	0.64	0.63	0.60	1.38	1.47
S3-6-7	18	0.66	0.64	0.63	0.60	1.20	1.28
S4-6-7	18	0.66	0.64	0.63	0.60	1.26	1.30
S5-6-7	18	0.66	0.64	0.63	0.60	1.23	1.35
R-12-7	36	1.10	1.07	1.17	1.20	1.76	1.95
S1-12-7	36	1.10	1.07	1.17	1.20	1.61	1.78
S2-12-7	36	1.10	1.07	1.17	1.20	1.62	1.81
S3-12-7	36	1.10	1.07	1.17	1.20	1.53	1.59
S4-12-7	36	1.10	1.07	1.17	1.20	1.46	1.61
S5-12-7	36	1.10	1.07	1.17	1.20	1.53	1.74
R-18-7	54	1.49	1.46	1.69	1.80	2.49	2.73
S1-18-7	54	1.49	1.46	1.69	1.80	2.28	2.58
S2-18-7	54	1.49	1.46	1.69	1.80	2.33	2.65
S3-18-7	54	1.49	1.46	1.69	1.80	2.05	2.35
S4-18-7	54	1.49	1.46	1.69	1.80	2.06	2.33
S5-18-7	54	1.49	1.46	1.69	1.80	2.25	2.62
R-24-7	72	1.85	1.81	2.19	2.40	2.55	2.97
S1-24-7	72	1.85	1.81	2.19	2.40	2.40	2.80
S2-24-7	72	1.85	1.81	2.19	2.40	2.48	2.91
S3-24-7	72	1.85	1.81	2.19	2.40	2.18	2.57
S4-24-7	72	1.85	1.81	2.19	2.40	2.39	2.54
S5-24-7	72	1.85	1.81	2.19	2.40	2.43	3.02
R-30-7	90	2.19	2.14	2.67	3.00	3.11	3.78
S1-30-7	90	2.19	2.14	2.67	3.00	2.84	3.44
S2-30-7	90	2.19	2.14	2.67	3.00	2.96	3.58
S3-30-7	90	2.19	2.14	2.67	3.00	2.60	3.17
S4-30-7	90	2.19	2.14	2.67	3.00	2.57	3.21
S5-30-7	90	2.19	2.14	2.67	3.00	3.06	3.74

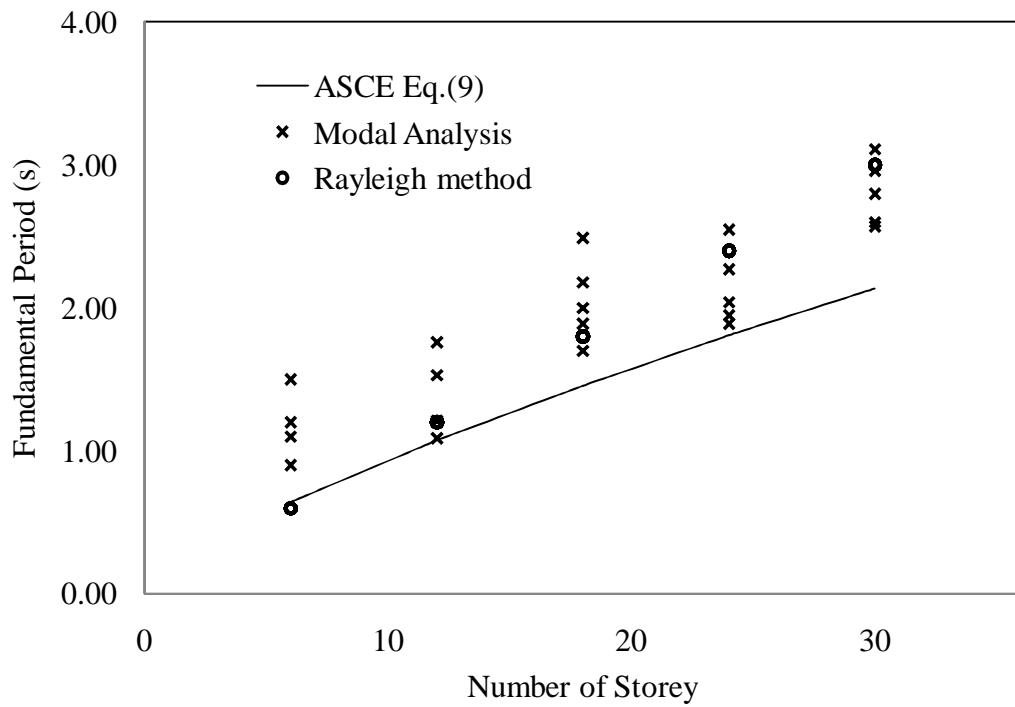
The results presented in Tables 4.1 – 4.3 are also shown graphically in Figs 4.1 - 4.3 for better understanding. The fundamental periods of 6 to 30 story setback buildings are plotted against number of stories. Fig. 4.1 presents the comparison of fundamental period of setback buildings with that obtained from IS 1893:2002 equation. This figure shows that the code empirical formula gives the lower-bound of the fundamental periods obtained from Modal Analysis and Raleigh Method. Therefore, it can be concluded that the code (IS 1893:2002) always gives conservative estimates of the fundamental periods of setback buildings with 6 to 30 storeys. It can also be seen that Raleigh Method underestimates the fundamental periods of setback buildings slightly which is also conservative for the selected buildings.



**Fig. 4.1:** Comparison of fundamental period of setback buildings with that obtained from IS 1893:2002 equation.



**Fig. 4.2:** Comparison of fundamental period of setback buildings with that obtained from ASCE 7:2010 Eq (2.8).



**Fig. 4.3:** Comparison of fundamental period of setback buildings with that obtained from ASCE 7:2010 Eq (2.9).



Figures 4.2 and 4.3 present the comparison of fundamental period of setback buildings with that obtained from Eqs. 2.8 and 2.9 of ASCE 7:2010 respectively. A similar conclusions to that of IS 1893:2002 can be made from the results presented in Figs. 4.2 - 4.3. This is to be noted that unlike other available equations, Eq. 2.9 from ASCE 7: 2010 does not consider the height of the building but it considers only the number of storeys of the buildings. Although this is not supported theoretically the Fig. 4.3 shows that this approach is most conservative among other code equations. It is instructive to note from these three figures that the fundamental period in a framed building is not a function of building height only. These three figures clearly show that buildings with same overall height may have different fundamental periods with a considerable variation which is not addressed in the code empirical equations.

As discussed in Section 2.3.1 the height of the building is not defined in the design code adequately. For a regular building there is no ambiguity as the height of the building is same throughout both the horizontal directions. However, this is not the case for setback buildings where building height may change from one end to other. Therefore, there is a need to define the irregularity for a setback building and relate the empirical equation of fundamental period of the setback building with its irregularity. Some of the previous works addressed this issue of defining irregularity and proposed some measure of quantifying the irregularity in setback buildings. Section 2.2 discusses these literatures in detail. Design codes do not directly quantify the irregularity in setback buildings but it gives a parameter to distinguish the regular and setback irregular buildings. These are discussed in Section 2.3.1 of Chapter 2.

The amounts of setback irregularity present in the selected buildings are calculated as per the definition given in the available literature as well as the international design codes and are presented in Tables 4.4 – 4.6.

**Table 4.4:** Characteristics of setback buildings with 5 m bay width

Building Designation	Height (m)	$T_{\text{Modal}}$ (s)	$\frac{A}{L}$ (IS 1893)	$\frac{L_{i+1}}{L_i}$ (ASCE 7)	Karavasilis <i>et.al.</i> 2008		$\eta$ (Sarkar <i>et.al.</i> 2010)
					$\phi_s$	$\phi_b$	
R-6-5	18	1.17	0.00	1.00	1.00	1.00	1.00
S1-6-5	18	1.05	0.33	1.50	1.25	1.25	0.75
S2-6-5	18	1.09	0.33	1.50	1.25	2.00	0.70
S3-6-5	18	0.95	0.66	2.00	1.75	1.75	0.65
S4-6-5	18	0.97	0.66	3.00	2.00	1.25	0.72
S5-6-5	18	1.01	0.66	3.00	2.00	2.00	0.55
R-12-5	36	1.49	0.00	1.00	1.00	1.00	1.00
S1-12-5	36	1.37	0.33	1.50	1.10	1.25	0.94
S2-12-5	36	1.4	0.33	1.50	1.10	2.00	0.85
S3-12-5	36	1.24	0.66	2.00	1.30	1.75	0.79
S4-12-5	36	1.24	0.66	3.00	1.40	1.25	0.88
S5-12-5	36	1.4	0.66	3.00	1.40	2.00	0.65
R-18-5	54	2.18	0.00	1.00	1.00	1.00	1.00
S1-18-5	54	2.00	0.33	1.50	1.03	1.25	0.94
S2-18-5	54	2.08	0.33	1.50	1.03	2.00	0.85
S3-18-5	54	1.84	0.66	2.00	1.09	1.75	0.78
S4-18-5	54	1.82	0.66	3.00	1.18	1.25	0.88
S5-18-5	54	2.16	0.66	3.00	1.18	2.00	0.64
R-24-5	72	2.44	0.00	1.00	1.00	1.00	1.00
S1-24-5	72	2.29	0.33	1.50	1.02	1.25	1.16
S2-24-5	72	2.43	0.33	1.50	1.02	2.00	1.01
S3-24-5	72	2.16	0.66	2.00	1.07	1.75	0.80
S4-24-5	72	2.09	0.66	3.00	1.09	1.25	1.07
S5-24-5	72	2.72	0.66	3.00	1.09	2.00	0.78
R-30-5	90	3.18	0.00	1.00	1.00	1.00	1.00
S1-30-5	90	2.89	0.33	1.50	1.02	1.25	0.94
S2-30-5	90	3.12	0.33	1.50	1.02	2.00	0.84
S3-30-5	90	2.76	0.66	2.00	1.05	1.75	0.76
S4-30-5	90	2.63	0.66	3.00	1.07	1.25	0.86
S5-30-5	90	3.55	0.66	3.00	1.07	2.00	0.62

**Table 4.5:** Characteristics of setback buildings with 6 m bay width

Building Designation	Height (m)	$T_{\text{Modal}}$ (s)	$\frac{A}{L}$ (IS 1893)	$\frac{L_{i+1}}{L_i}$ (ASCE 7)	Karavasilis <i>et.al.</i> 2008		$\eta$ (Sarkar <i>et.al.</i> 2010)
					$\phi_s$	$\phi_b$	
R-6-6	18	1.37	0.00	1.00	1.00	1.00	1.00
S1-6-6	18	1.23	0.33	1.50	1.25	1.25	0.79
S2-6-6	18	1.28	0.33	1.50	1.25	2.00	0.73
S3-6-6	18	1.11	0.66	2.00	1.75	1.75	0.67
S4-6-6	18	1.13	0.66	3.00	2.00	1.25	0.75
S5-6-6	18	1.17	0.66	3.00	2.00	2.00	0.57
R-12-6	36	1.72	0.00	1.00	1.00	1.00	1.00
S1-12-6	36	1.57	0.33	1.50	1.10	1.25	0.95
S2-12-6	36	1.60	0.33	1.50	1.10	2.00	0.85
S3-12-6	36	1.41	0.66	2.00	1.30	1.75	0.79
S4-12-6	36	1.42	0.66	3.00	1.40	1.25	0.88
S5-12-6	36	1.56	0.66	3.00	1.40	2.00	0.66
R-18-6	54	2.45	0.00	1.00	1.00	1.00	1.00
S1-18-6	54	2.28	0.33	1.50	1.03	1.25	0.96
S2-18-6	54	2.35	0.33	1.50	1.03	2.00	0.86
S3-18-6	54	2.08	0.66	2.00	1.09	1.75	0.78
S4-18-6	54	2.06	0.66	3.00	1.18	1.25	0.89
S5-18-6	54	2.37	0.66	3.00	1.18	2.00	0.66
R-24-6	72	2.68	0.00	1.00	1.00	1.00	1.00
S1-24-6	72	2.52	0.33	1.50	1.02	1.25	0.69
S2-24-6	72	2.65	0.33	1.50	1.02	2.00	0.62
S3-24-6	72	2.35	0.66	2.00	1.07	1.75	0.56
S4-24-6	72	2.3	0.66	3.00	1.09	1.25	0.64
S5-24-6	72	2.84	0.66	3.00	1.09	2.00	0.47
R-30-6	90	3.45	0.00	1.00	1.00	1.00	1.00
S1-30-6	90	3.19	0.33	1.50	1.02	1.25	0.96
S2-30-6	90	3.32	0.33	1.50	1.02	2.00	0.86
S3-30-6	90	2.94	0.66	2.00	1.05	1.75	0.78
S4-30-6	90	2.84	0.66	3.00	1.07	1.25	0.88
S5-30-6	90	3.64	0.66	3.00	1.07	2.00	0.64

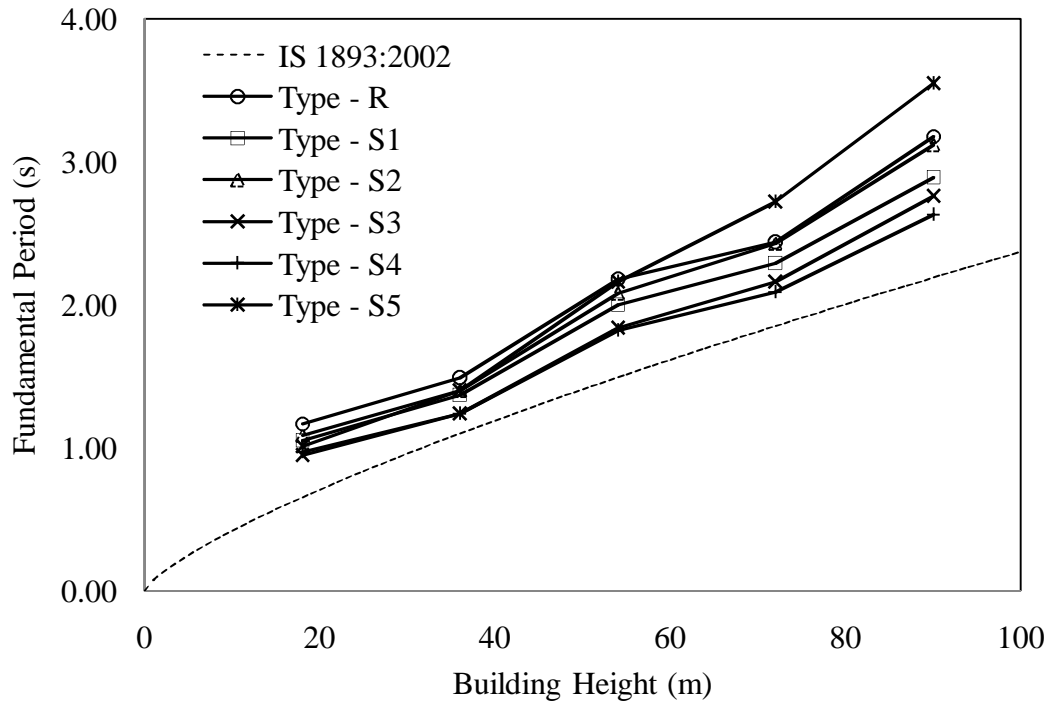
**Table 4.6:** Characteristics of setback buildings with 7 m bay width

Building Designation	Height (m)	$T_{\text{Modal}}$ (s)	$\frac{A}{L}$ (IS 1893)	$\frac{L_{i+1}}{L_i}$ (ASCE 7)	Karavasilis <i>et.al.</i> 2008		$\eta$ (Sarkar <i>et.al.</i> 2010)
					$\phi_s$	$\phi_b$	
R-6-7	18	1.58	0.00	1.00	1.00	1.00	1.00
S1-6-7	18	1.42	0.33	1.50	1.25	1.25	0.86
S2-6-7	18	1.47	0.33	1.50	1.25	2.00	0.80
S3-6-7	18	1.28	0.66	2.00	1.75	1.75	0.74
S4-6-7	18	1.30	0.66	3.00	2.00	1.25	0.82
S5-6-7	18	1.35	0.66	3.00	2.00	2.00	0.63
R-12-7	36	1.95	0.00	1.00	1.00	1.00	1.00
S1-12-7	36	1.78	0.33	1.50	1.10	1.25	0.94
S2-12-7	36	1.81	0.33	1.50	1.10	2.00	0.85
S3-12-7	36	1.59	0.66	2.00	1.30	1.75	0.79
S4-12-7	36	1.61	0.66	3.00	1.40	1.25	0.88
S5-12-7	36	1.74	0.66	3.00	1.40	2.00	0.66
R-18-7	54	2.73	0.00	1.00	1.00	1.00	1.00
S1-18-7	54	2.58	0.33	1.50	1.03	1.25	0.97
S2-18-7	54	2.65	0.33	1.50	1.03	2.00	0.88
S3-18-7	54	2.35	0.66	2.00	1.09	1.75	0.81
S4-18-7	54	2.33	0.66	3.00	1.18	1.25	0.91
S5-18-7	54	2.62	0.66	3.00	1.18	2.00	0.67
R-24-7	72	2.97	0.00	1.00	1.00	1.00	1.00
S1-24-7	72	2.80	0.33	1.50	1.02	1.25	0.92
S2-24-7	72	2.91	0.33	1.50	1.02	2.00	0.83
S3-24-7	72	2.57	0.66	2.00	1.07	1.75	0.76
S4-24-7	72	2.54	0.66	3.00	1.09	1.25	0.85
S5-24-7	72	3.02	0.66	3.00	1.09	2.00	0.63
R-30-7	90	3.78	0.00	1.00	1.00	1.00	1.00
S1-30-7	90	3.44	0.33	1.50	1.02	1.25	0.94
S2-30-7	90	3.58	0.33	1.50	1.02	2.00	0.84
S3-30-7	90	3.17	0.66	2.00	1.05	1.75	0.76
S4-30-7	90	3.21	0.66	3.00	1.07	1.25	0.86
S5-30-7	90	3.74	0.66	3.00	1.07	2.00	0.62

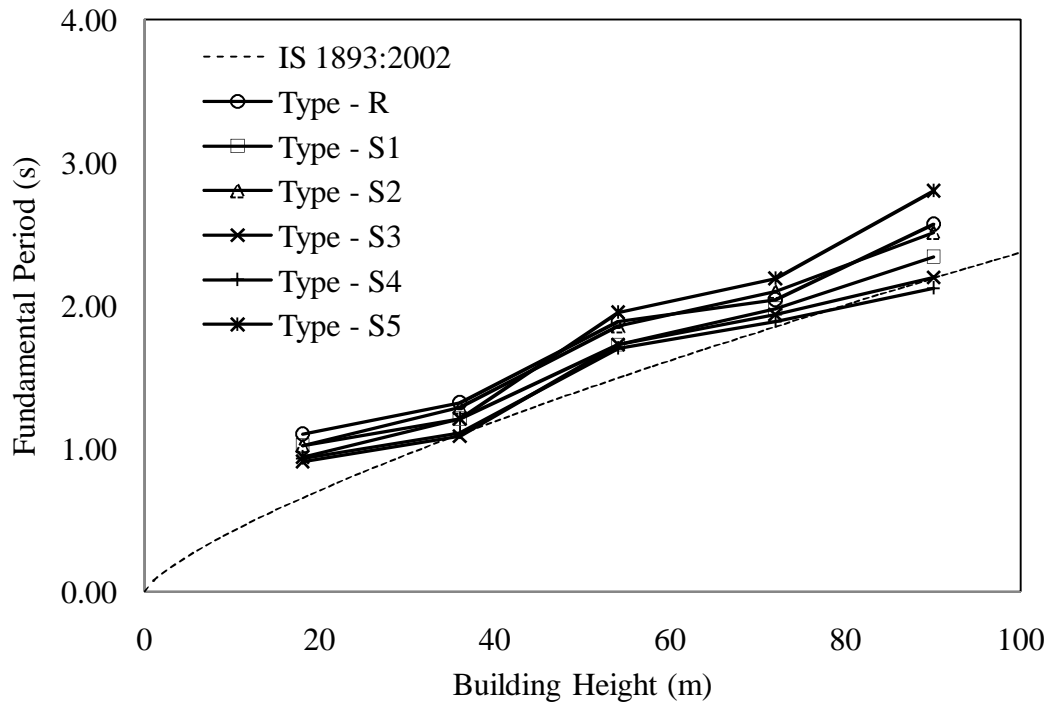
Table 4.4 presents the results of buildings with 5m bay width, Table 4.5 presents the results of buildings with 6m bay width whereas the Table 4.6 presents the results of buildings with 7m bay width. The height of the building presented here are maximum height of the buildings. The fundamental periods presented here are obtained from modal analysis.

It can be seen from these tables that the parameter given in IS 1893 and ASCE 7 to distinguish setback irregularity are quite similar yielding similar results except for few buildings. One of the two indices ( $\phi_b$ ) given by Karavasilis et. al., 2008 is an improved version of that presented in ASCE 7 where it considers the summation of variation of building width along its height instead of variation of building width in one adjacent floor. Sarkar et. al. (2010) defined the irregularity in terms of the modal parameters. This procedure is based on two-dimensional plane frame analysis. While calculating the regularity index using this method, it is found to be not suitable for a three dimensional building. Fundamental mode vibration of a setback building and a similar regular building may not be in the same horizontal direction for a three dimensional building and it is difficult to use this method for such buildings. Also, it is clear from these three tables presented above that the change in period due to the setback irregularity is not consistent with any of these parameters discussed here.

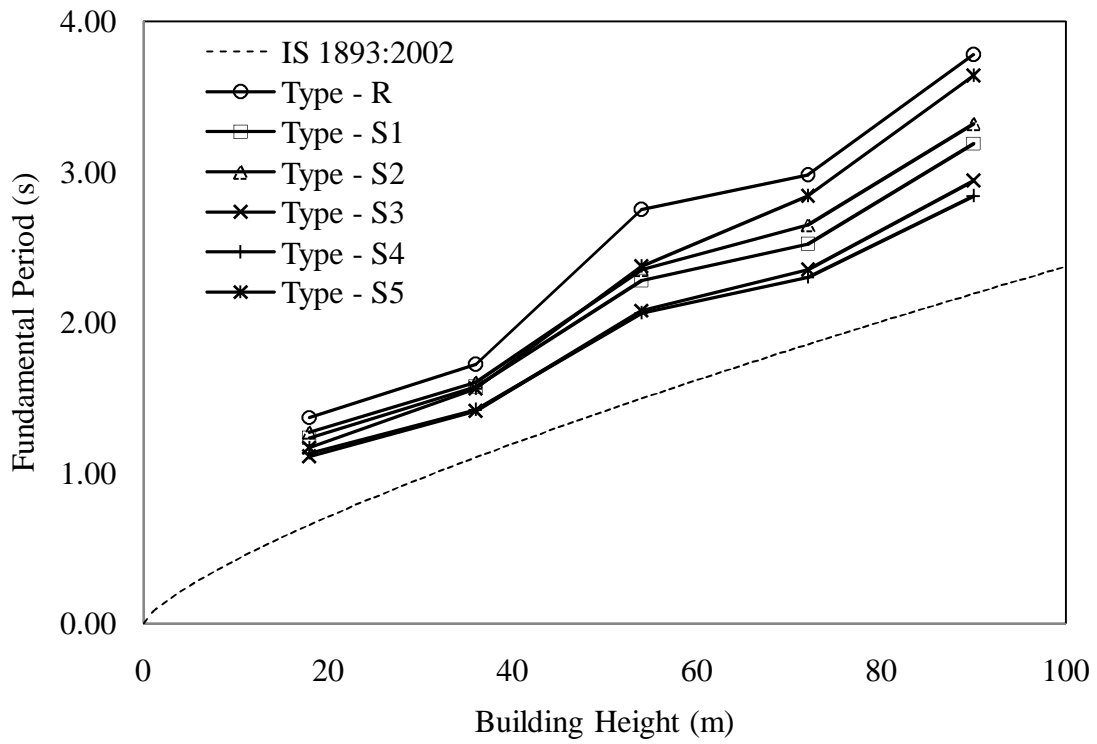
Fundamental period for different setback buildings are shown in Figs.4.4 - 4.9 as a function of maximum building height. Fundamental periods obtained from Modal analyses and Rayleigh analyses are plotted separately and are compared with that obtained from IS 1893:2002 empirical equation. Fundamental period of all the setback types (S1 to S5) along with regular (R) buildings are shown in a single plot so as to analyse the pattern of variation of fundamental period. The results obtained from ASCE 7: 2010 are found to be similar to those obtained from IS 1893:2002 hence not shown separately.



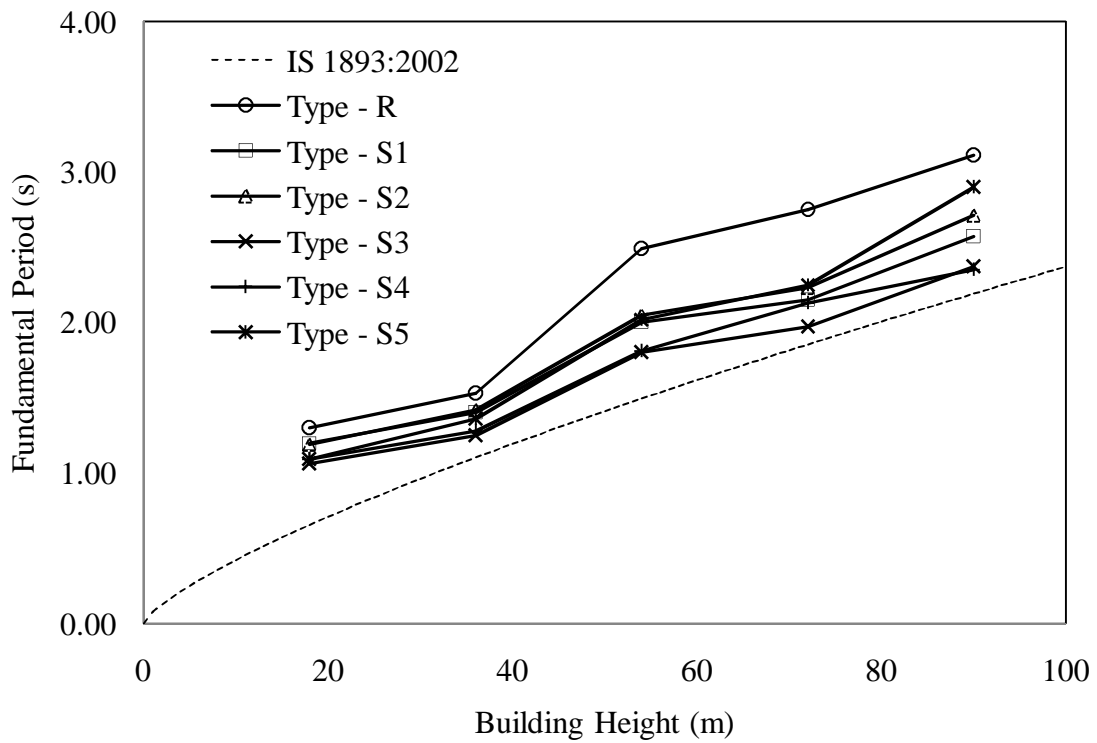
**Fig. 4.4:** Fundamental period (Modal) versus height of setback buildings of 5m bay width



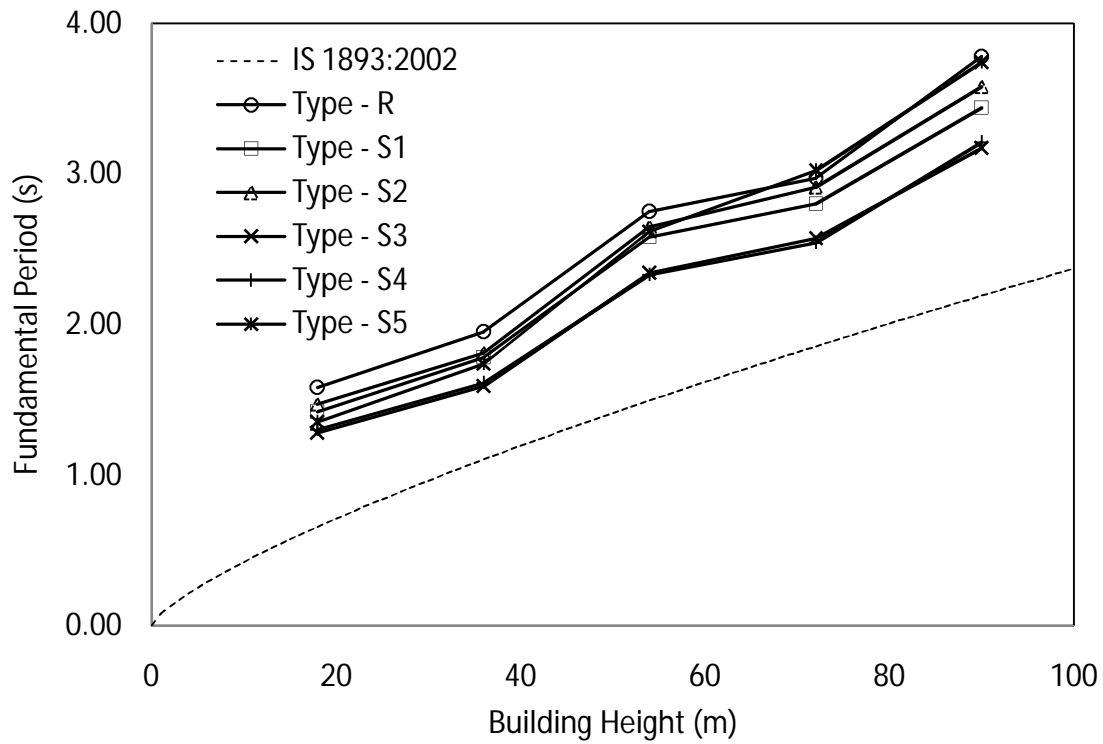
**Fig. 4.5:** Fundamental period (Rayleigh) versus height of setback buildings of 5m bay width



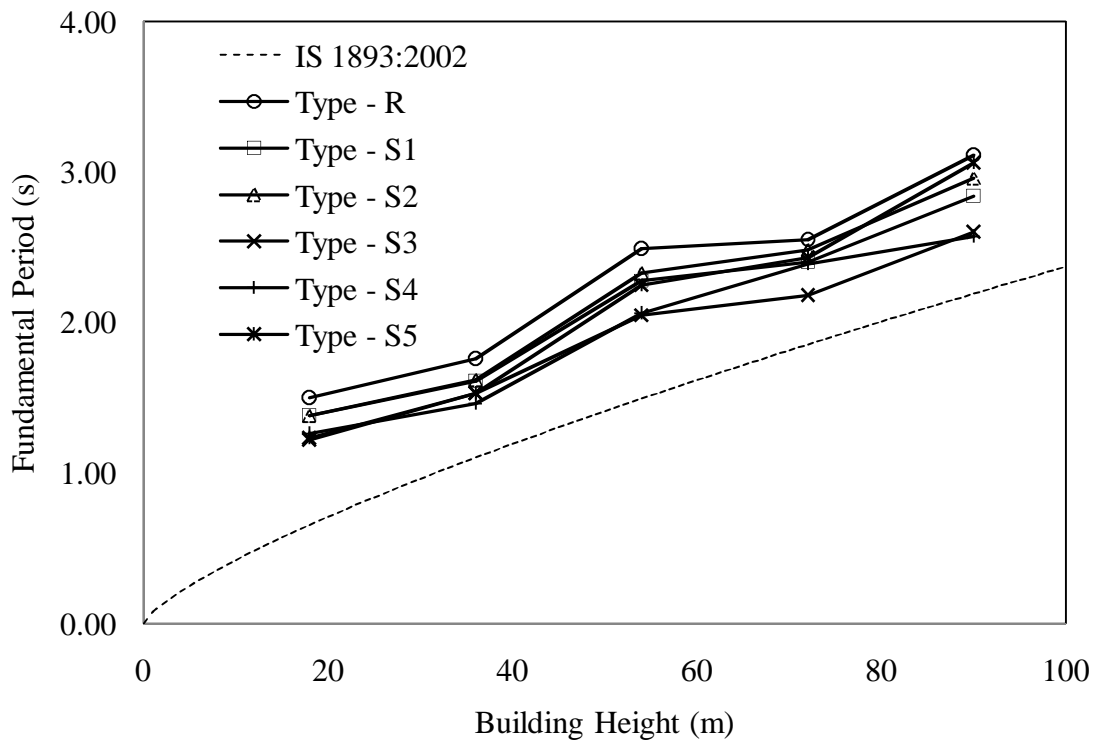
**Fig. 4.6:** Fundamental period (Modal) versus height of setback buildings of 6m bay width



**Fig. 4.7:** Fundamental period (Rayleigh) versus height of setback buildings of 6m bay width



**Fig. 4.8:** Modal analysis time period versus height of setback buildings of 7m bay width



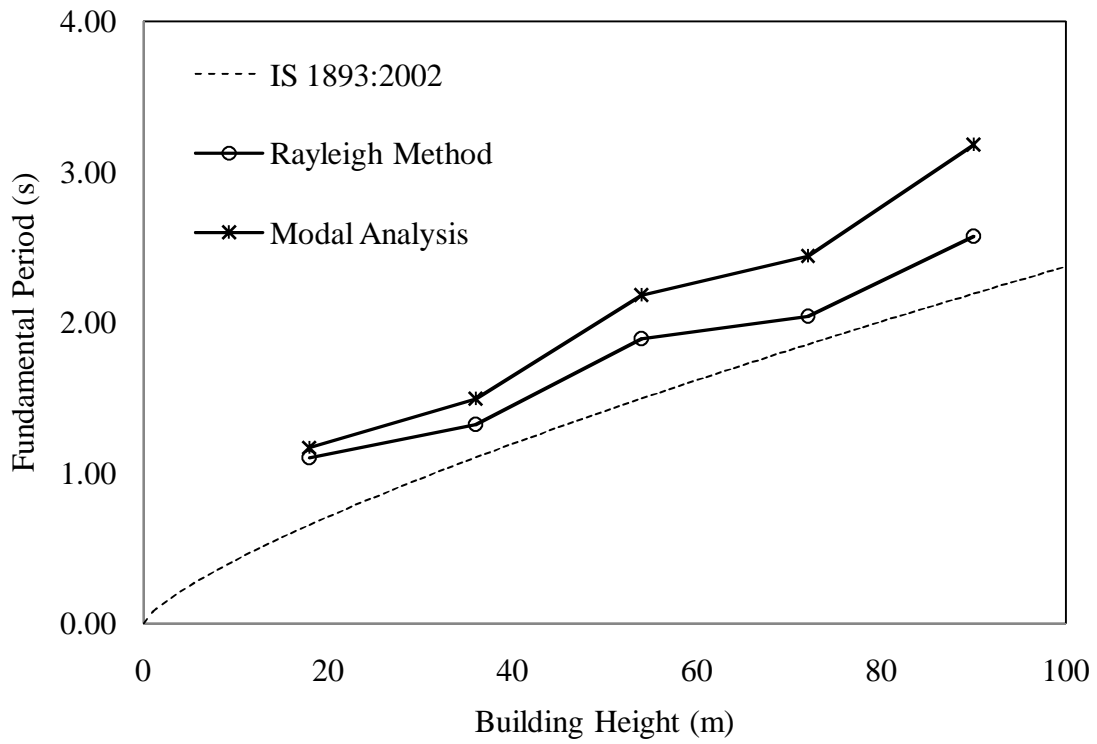
**Fig. 4.9:** Rayleigh analysis time period versus height of setback buildings of 7m bay width



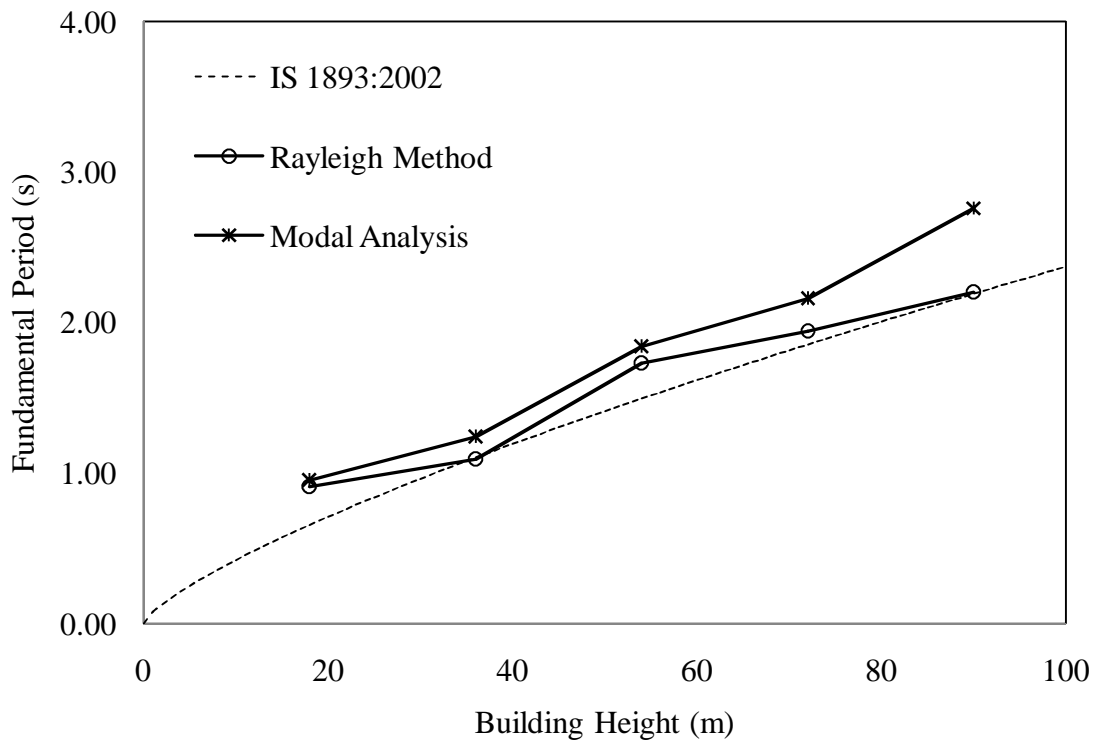
Figs.4.4 - 4.9 presented above show that the buildings with same maximum height and same maximum width may have different period depending on the amount of irregularity present in the setback buildings. This variation of the fundamental periods due to variation in irregularity is found to be more for taller buildings and comparatively less for shorter buildings. This observation is valid for the periods calculated from both modal and Rayleigh analysis. It is found that variation of fundamental periods calculated from modal analysis and Rayleigh method are quite similar.

#### **4.2.2 PARAMETERS AFFECTING FUNDAMENTAL TIME PERIOD**

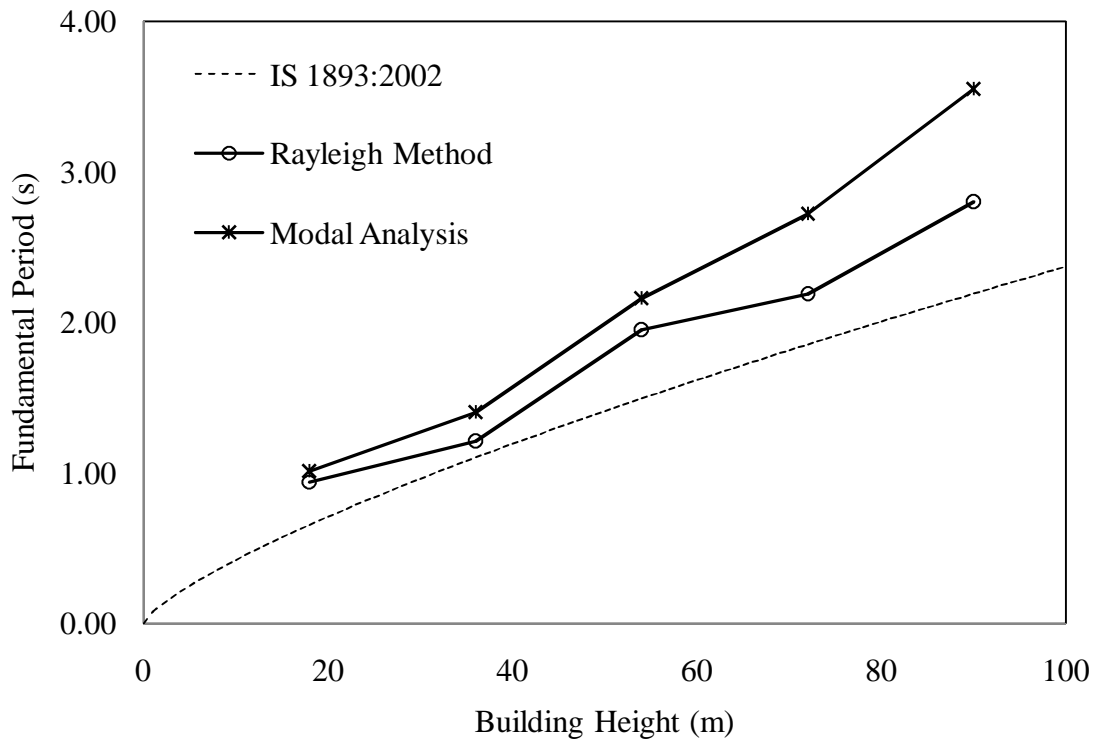
One of the main objectives of the present study was to formulate an improved empirical relation to evaluate fundamental period of setback buildings considering the vertical geometric irregularity. It is, therefore, required to know the important parameters which control the fundamental period of a setback building. This section analyses the fundamental period computed using the Rayleigh method and Modal analysis against different possible parameters. Although the results of all the selected buildings are considered for analysis, results of 15 building are presented here for convenience. Figs. 4.10-4.12 present the fundamental periods of three irregular building variants as a function of height keeping bay width same. This figure shows that the fundamental period is indeed very sensitive to the building height. Figs. 4.13 – 4.15 present the fundamental periods of three irregular building variants as a function of bay width keeping the building height same. Figs. 4.16



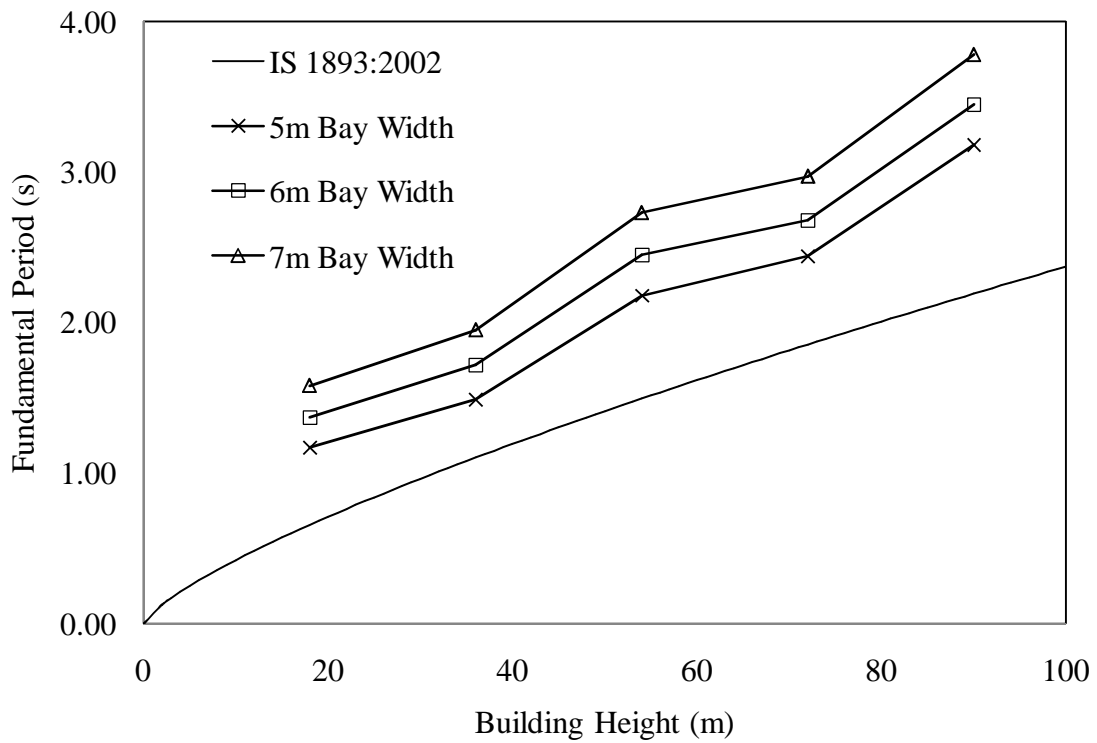
**Fig. 4.10:** Fundamental time period vs. height of Type - R building with 5 m bay width



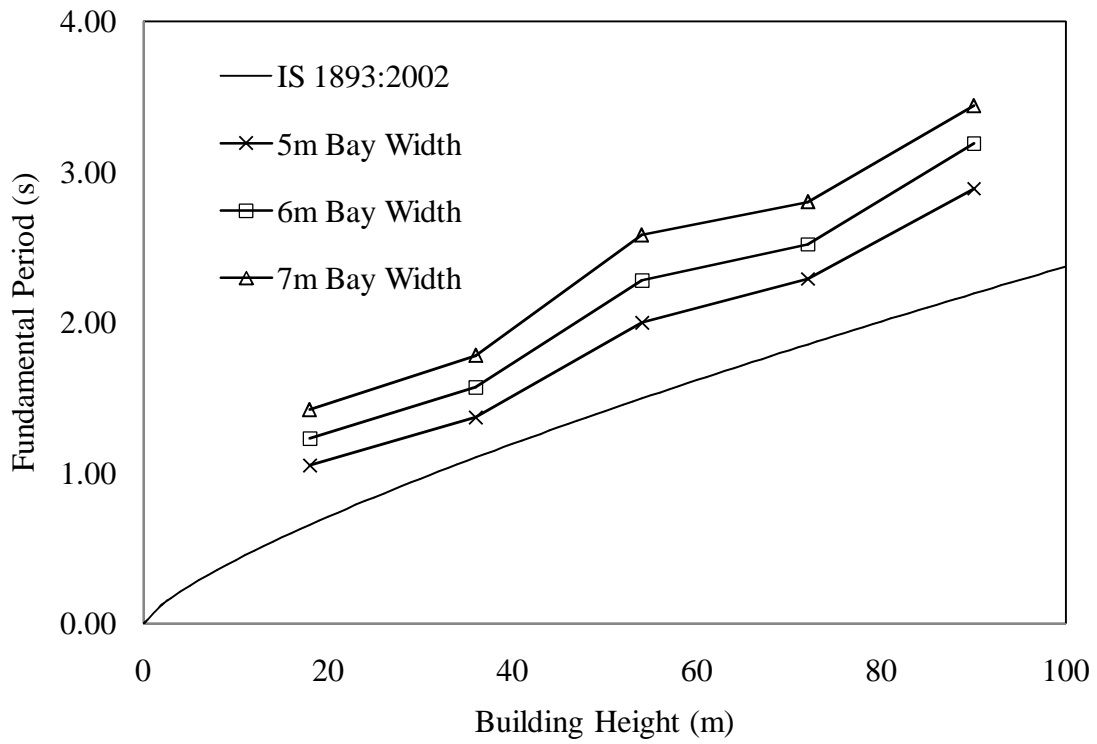
**Fig. 4.11:** Fundamental time period vs. height of Type-S3 setback building with 5 m bay width



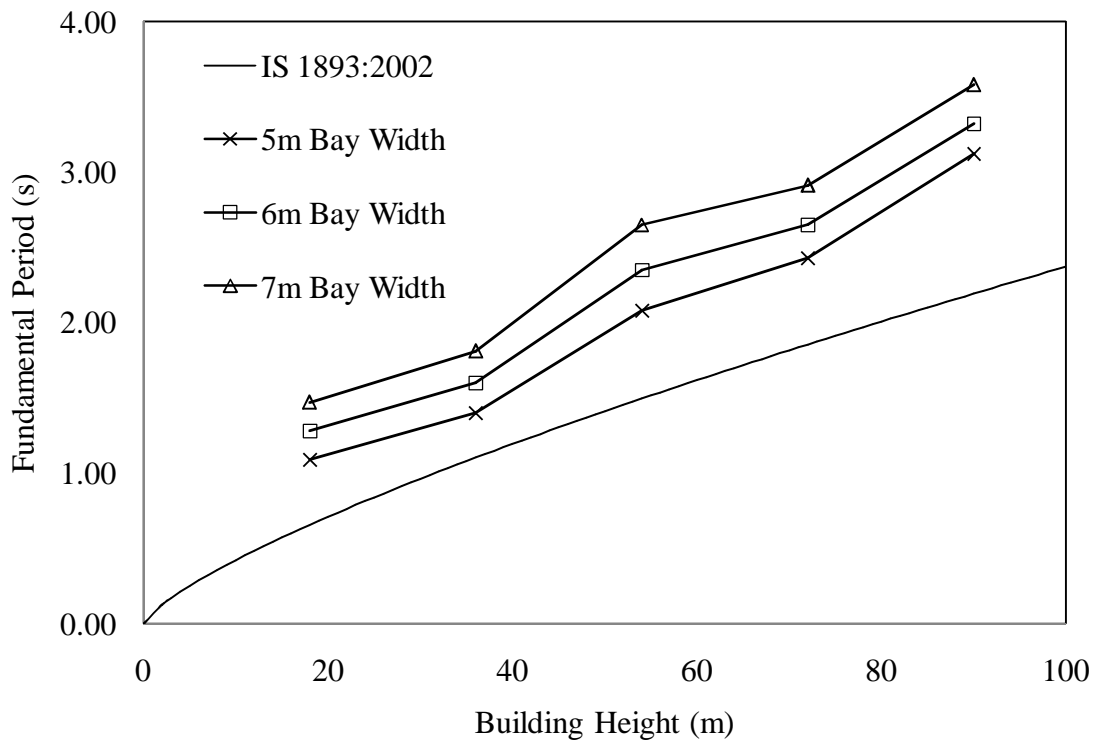
**Fig. 4.12:** Fundamental time period vs. height of Type-S5 setback building with 5 m bay width



**Fig. 4.13:** Variation of fundamental time period with bay width for Type-R building.



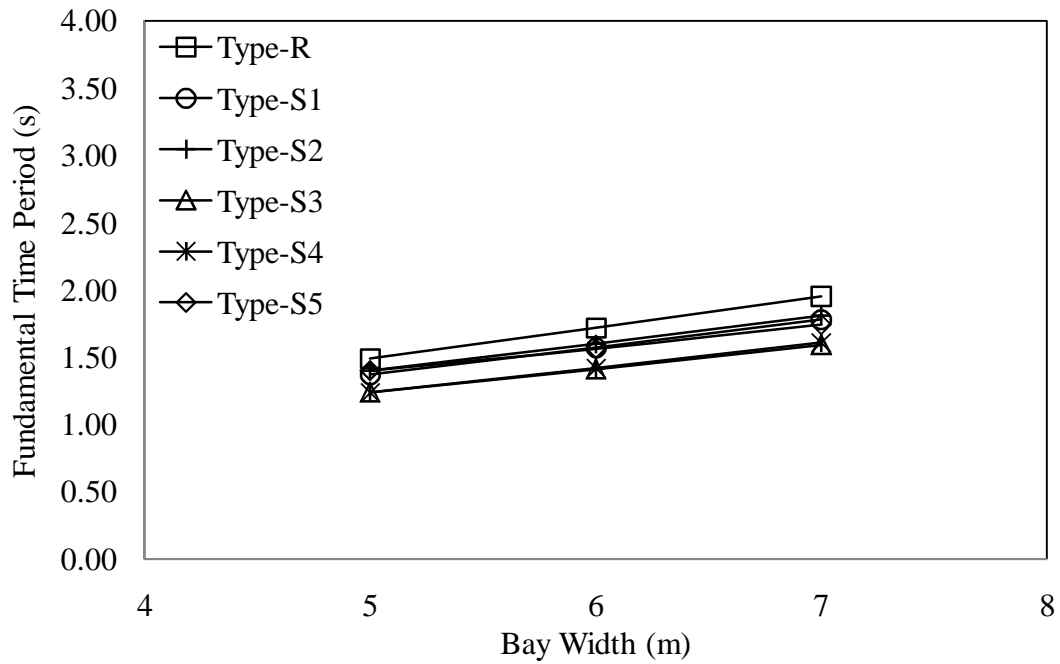
**Fig. 4.14:** Variation of fundamental time period with bay width for Type – S1 setback building.



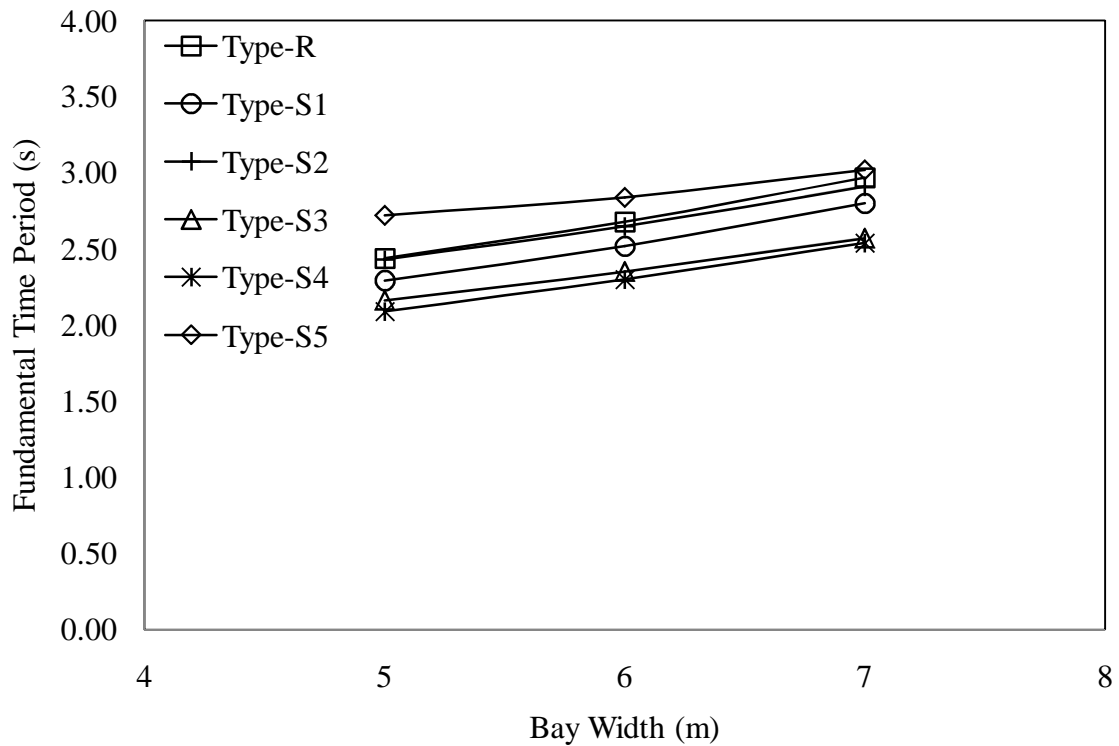
**Fig. 4.15:** Variation of fundamental time period with bay width for Type – S2 setback building

All the major international design codes including IS 1893:2002 does not specify bay width or plan dimension as a parameter which affects the fundamental period of RC framed building without considering brick infill. However, it is observed that the bay width or the plan dimension affects the fundamental period of such type of buildings. Figs.4.16 - 4.17 presents the variation in fundamental period with the change in bay width of the setback building, it is observed from these figures that, the change in bay width affects the fundamental period of the setback building considerably.

Fig 4.16 and 4.17 presents the variation of fundamental time period with bay width for 12 storey setback building and 24 storey setback buildings. This change in fundamental period due to change in bay width is found to be considerable and it cannot be ignored. The code based empirical equation for the estimation of fundamental period does not take in account the bay width of the building for RC moment resisting frames without brick infill. However, in design codes, the empirical equations considering the brick infill does depend on bay width. Therefore it is concluded that the bay width or the plan dimension of the building affects the fundamental period of building, and it should be accounted for in the code based empirical equations for the calculation of fundamental period of RC frame buildings without infill also.



**Fig. 4.16:** Variation of fundamental time period with bay width for 12-storey setback buildings.



**Fig. 4.17** Variation of fundamental time period with bay width for 24-storey setback buildings

Section 4.2.1 explained that the fundamental period is also sensitive to the setback irregularity of the buildings. As explained earlier the measures to quantify the irregularity given in literatures are found to be not very efficient as a parameter for formulation. Therefore, a new approach of considering average height and average width of the setback buildings was tried to define the irregularity in line with Young (2011). The average height is calculated as the ratio of summation of the heights of individual bay to the number of bays. Similarly the average width is calculated as the ratio of summation of the width of the individual storey to the number of storeys. These average height and average width made non-dimensional with respect to maximum building height and maximum building width at base, respectively.

Tables 4.7 - 4.9 present the details of normalised average height and normalised average width of all the selected buildings. The fundamental period of the corresponding building also presented to correlate them. It is interesting to see from the Tables 4.7 - 4.9 that the normalised average height and normalised average width for any setback building is same. Also, these tables show that fundamental period of the regular building is always more than that of setback buildings. However, the fundamental periods of setback buildings are not consistent with the normalised average height or width of the buildings. Fig. 4.16 presents the fundamental period scatter of the setback buildings against the normalised average height/width of the buildings. This figure clearly shows that there is hardly any correlation between normalised average height/width and the fundamental period of setback buildings.

**Table 4.7:** Normalised average height and width of the buildings with 5m bay width

Building Designation	$\frac{h_{av}}{h}$	$\frac{d_{av}}{d}$	Fundamental Period
R-6-5	1.00	1.00	1.17
S1-6-5	0.89	0.89	1.05
S2-6-5	0.78	0.78	1.09
S3-6-5	0.67	0.67	0.95
S4-6-5	0.78	0.78	0.97
S5-6-5	0.56	0.56	1.01
R-12-5	1.00	1.00	1.49
S1-12-5	0.89	0.89	1.37
S2-12-5	0.78	0.78	1.40
S3-12-5	0.67	0.67	1.24
S4-12-5	0.78	0.78	1.24
S5-12-5	0.56	0.56	1.40
R-18-5	1.00	1.00	2.18
S1-18-5	0.89	0.89	2.00
S2-18-5	0.78	0.78	2.08
S3-18-5	0.67	0.67	1.84
S4-18-5	0.78	0.78	1.82
S5-18-5	0.56	0.56	2.16
R-24-5	1.00	1.00	2.44
S1-24-5	0.89	0.89	2.29
S2-24-5	0.78	0.78	2.43
S3-24-5	0.67	0.67	2.16
S4-24-5	0.78	0.78	2.09
S5-24-5	0.56	0.56	2.72
R-30-5	1.00	1.00	3.18
S1-30-5	0.89	0.89	2.89
S2-30-5	0.78	0.78	3.12
S3-30-5	0.67	0.67	2.76
S4-30-5	0.78	0.78	2.63
S5-30-5	0.56	0.56	3.55

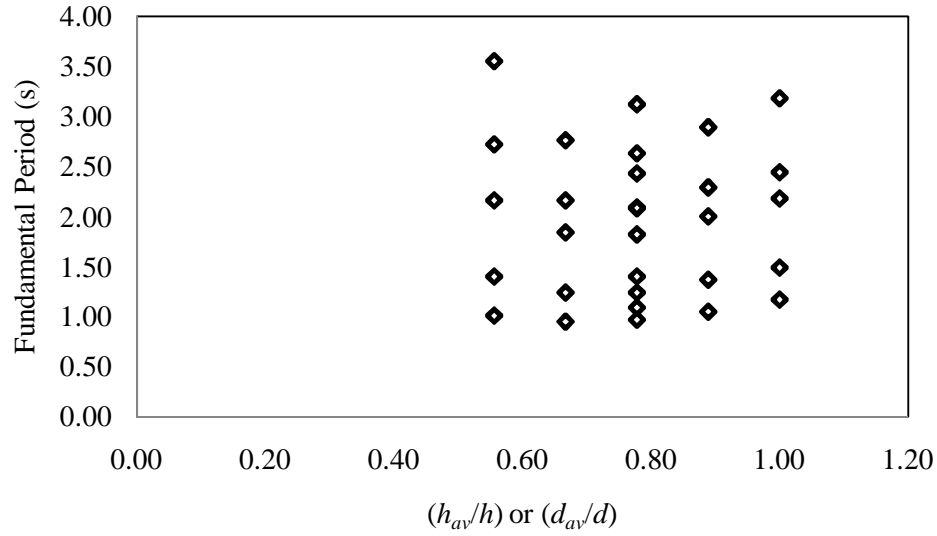


**Table 4.8:** Normalised average height and width of the buildings with 6m bay width

Building Designation	$\frac{h_{av}}{h}$	$\frac{d_{av}}{d}$	Fundamental Period
R-6-6	1.00	1.00	1.37
S1-6-6	0.89	0.89	1.23
S2-6-6	0.78	0.78	1.28
S3-6-6	0.67	0.67	1.11
S4-6-6	0.78	0.78	1.13
S5-6-6	0.56	0.56	1.17
R-12-6	1.00	1.00	1.72
S1-12-6	0.89	0.89	1.57
S2-12-6	0.78	0.78	1.60
S3-12-6	0.67	0.67	1.41
S4-12-6	0.78	0.78	1.42
S5-12-6	0.56	0.56	1.56
R-18-6	1.00	1.00	2.45
S1-18-6	0.89	0.89	2.28
S2-18-6	0.78	0.78	2.35
S3-18-6	0.67	0.67	2.08
S4-18-6	0.78	0.78	2.06
S5-18-6	0.56	0.56	2.37
R-24-6	1.00	1.00	2.68
S1-24-6	0.89	0.89	2.52
S2-24-6	0.78	0.78	2.65
S3-24-6	0.67	0.67	2.35
S4-24-6	0.78	0.78	2.30
S5-24-6	0.56	0.56	2.84
R-30-6	1.00	1.00	3.45
S1-30-6	0.89	0.89	3.19
S2-30-6	0.78	0.78	3.32
S3-30-6	0.67	0.67	2.94
S4-30-6	0.78	0.78	2.84
S5-30-6	0.56	0.56	3.64

**Table 4.9:** Normalised average height and width of the buildings with 7m bay width

Building Designation	$\frac{h_{av}}{h}$	$\frac{d_{av}}{d}$	Fundamental Period
R-6-7	1.00	1.00	1.58
S1-6-7	0.89	0.89	1.42
S2-6-7	0.78	0.78	1.47
S3-6-7	0.67	0.67	1.28
S4-6-7	0.78	0.78	1.30
S5-6-7	0.56	0.56	1.35
R-12-7	1.00	1.00	1.95
S1-12-7	0.89	0.89	1.78
S2-12-7	0.78	0.78	1.81
S3-12-7	0.67	0.67	1.59
S4-12-7	0.78	0.78	1.61
S5-12-7	0.56	0.56	1.74
R-18-7	1.00	1.00	2.73
S1-18-7	0.89	0.89	2.58
S2-18-7	0.78	0.78	2.65
S3-18-7	0.67	0.67	2.35
S4-18-7	0.78	0.78	2.33
S5-18-7	0.56	0.56	2.62
R-24-7	1.00	1.00	2.97
S1-24-7	0.89	0.89	2.8
S2-24-7	0.78	0.78	2.91
S3-24-7	0.67	0.67	2.57
S4-24-7	0.78	0.78	2.54
S5-24-7	0.56	0.56	3.02
R-30-7	1.00	1.00	3.78
S1-30-7	0.89	0.89	3.44
S2-30-7	0.78	0.78	3.58
S3-30-7	0.67	0.67	3.17
S4-30-7	0.78	0.78	3.21
S5-30-7	0.56	0.56	3.74



**Fig. 4.18:** Fundamental period scatter against the normalised average height/width

### 4.3 SUMMARY

Fundamental period of all the selected building models were estimated as per modal analysis, Rayleigh method and empirical equations given in the design codes. The results were critically analysed and presented in this chapter. The aim of the analyses and discussions were to identify a parameter that describes the irregularity of a setback building and arrive at an improved empirical equation to estimate the fundamental period of setback buildings with confidence. However, this study shows that it is difficult to quantify the irregularity in a setback building with any single parameter. This study indicates that there is very poor correlation between fundamental periods of three dimensional buildings with any of the parameters used to define the setback irregularity by the previous researchers or design codes. However, it requires further investigation to arrive at a single or multiple parameters to accurately define the irregularity in a three dimensional setback buildings.

## CHAPTER 5

### SUMMARY AND CONCLUSIONS

#### 5.1 SUMMARY

The behaviour of a multi-storey framed building during strong earthquake motions depends on the distribution of mass, stiffness, and strength in both the horizontal and vertical planes of a building. In multi-storeyed framed buildings, damage from earthquake ground motion generally initiates at locations of structural weaknesses present in the lateral load resisting frames. Further, these weaknesses tend to accentuate and concentrate the structural damage through plastification that eventually leads to complete collapse. In some cases, these weaknesses may be created by discontinuities in stiffness, strength or mass between adjacent storeys. Such discontinuities between storeys are often associated with sudden variations in the frame geometry along the height. There are many examples of failure of buildings in past earthquakes due to such vertical discontinuities. Structural engineers have developed confidence in the design of buildings in which the distributions of mass, stiffness and strength are more or less uniform. But there is a less confidence about the design of structures having irregular geometrical configurations.

A common type of vertical geometrical irregularity in building structures arises is the presence of setbacks, *i.e.* the presence of abrupt reduction of the lateral dimension of the building at specific levels of the elevation. This building category is known as ‘setback building’. This building form is becoming increasingly popular in modern multi-storey

building construction mainly because of its functional and aesthetic architecture. In particular, such a setback form provides for adequate daylight and ventilation for the lower storeys in an urban locality with closely spaced tall buildings. This type of building form also provides for compliance with building bye-law restrictions related to 'floor area ratio' (practice in India). Setback buildings are characterised by staggered abrupt reductions in floor area along the height of the building, with consequent drops in mass, strength and stiffness. This setback affects the mass, strength, stiffness, centre of mass and centre of stiffness of setback building. Dynamic characteristics of such buildings differ from the regular building due to changes in geometrical and structural property. Design codes are not clear about the definition of building height for computation of fundamental period. The bay-wise variation of height in setback building makes it difficult to compute natural period of such buildings.

With this background it is found essential to study the effect of setbacks on the fundamental period of buildings. Also, the performance of the empirical equation given in Indian Standard IS 1893:2002 for estimation of fundamental period of setback buildings is matter of concern for structural engineers.

To get a clear idea about the dynamic performance of setback buildings a detailed literature review is carried out in two major areas. These are: (i) Response of setback buildings under seismic loading, effect of vertical irregularity on fundamental period of building and the quantification of setback and (ii) the recommendations proposed by seismic design codes on setback buildings. The research papers on setback buildings conclude that the displacement demand is dependent on the geometrical configuration of frame and concentrated in the neighbourhood of the setbacks for setback structures. The

higher modes significantly contribute to the response quantities of structure. Empirical equations used in design codes, such as IS 1893:2002, ASCE 7:2010, Euro code 8 and Rayleigh method for the estimation of Fundamental period are discussed with reference to setback buildings. The different code recommendations for the description and quantification of irregular buildings are also discussed briefly. The applicability of code based empirical formulas for calculation of fundamental period of setback buildings was nowhere mentioned in the literature, except Sarkar *et. al.* (2010). The procedure discussed in this literature is based on two-dimensional plane frame analysis and not suitable for a realistic three dimensional building. Therefore, it is essential to develop an improvement in the code based empirical equation to estimate the fundamental period of setback buildings.

To achieve the objective of the study altogether 90 building frames were selected for the study representing the realistic three dimensional buildings of 6-30 storeys. Different building geometries were taken for the study. These building geometries represent varying degree of irregularity or amount of setback. Three different bay widths, i.e. 5m, 6m and 7m (in both the horizontal direction) with a uniform three number of bays at base were considered for this study. It should be noted that bay width of 4m – 7m is the usual case, especially in Indian and European practice. Similarly, five different height categories were considered for the study, ranging from 6 to 30 storeys, with a uniform storey height of 3m. Altogether 90 building frames with different amount of setback irregularities due to the reduction in width and height were selected. The building geometries considered in the present study are taken from literature (Karavasisis *et. al.*, 2008). The regular frame, without any setback, is also studied.

There are altogether six different building geometries, one regular and five irregular, for each height category are considered in the present study. The buildings are three dimensional, with the irregularity in the direction of setback, in the other horizontal direction the building is just repeating its geometric configuration. Setback frames are named as S1, S2, S3, S4 and S5 depending on the percentage reduction of floor area and height. The frames are designed with M-20 grade of concrete and Fe-415 grade of reinforcing steel as per prevailing Indian Standards. Gravity (dead and imposed) load and seismic load corresponding to seismic zone II of IS 1893:2002 are considered for the design.

All the selected building models with different setback irregularities are analyzed for linear dynamic behaviour using commercial software SAP2000 (v12) with a focus on fundamental time period.

## **5.2 CONCLUSIONS**

Fundamental period of all the selected building models were estimated as per modal analysis, Rayleigh method and empirical equations given in the design codes. The results were critically analysed and presented in this chapter. The aim of the analyses and discussions were to identify a parameter that describes the irregularity of a setback building and arrive at an improved empirical equation to estimate the fundamental period of setback buildings with confidence. However, this study shows that it is difficult to quantify the irregularity in a setback building with any single parameter. This study indicates that there is very poor correlation between fundamental periods of three dimensional buildings with any of the parameters used to define the setback irregularity

by the previous researchers or design codes. However, it requires further investigation to arrive at single or multiple parameters to accurately define the irregularity in a three dimensional setback buildings. Based on the work presented in this thesis following point-wise conclusions can be drawn:

- i) Period of setback buildings are found to be always less than that of similar regular building. Fundamental period of setback buildings are found to be varying with irregularity even if the height remain constant. The change in period due to the setback irregularity is not consistent with any of these parameters used in literature or design codes to define irregularity.
- ii) The code (IS 1893:2002) empirical formula gives the lower-bound of the fundamental periods obtained from Modal Analysis and Raleigh Method. Therefore, it can be concluded that the code (IS 1893:2002) always gives conservative estimates of the fundamental periods of setback buildings with 6 to 30 storeys. It can also be seen that Raleigh Method underestimates the fundamental periods of setback buildings slightly which is also conservative for the selected buildings. However the degree of conservativeness in setback building is not proportionate to that of regular buildings.
- iii) Unlike other available equations, Eq. 2.9 from ASCE 7: 2010 does not consider the height of the building but it considers only the number of storeys of the buildings. Although this is not supported theoretically this approach is found to be most conservative among other code equations.
- iv) It is found that the fundamental period in a framed building is not a function of building height only. This study shows that buildings with same overall



height may have different fundamental periods with a considerable variation which is not addressed in the code empirical equations.

- v) In the empirical equation of fundamental period, the height of the building is not defined in the design code adequately. For a regular building there is no ambiguity as the height of the building is same throughout both the horizontal directions. However, this is not the case for setback buildings where building height may change from one end to other.
- vi) The buildings with same maximum height and same maximum width may have different period depending on the amount of irregularity present in the setback buildings. This variation of the fundamental periods due to variation in irregularity is found to be more for taller buildings and comparatively less for shorter buildings. This observation is valid for the periods calculated from both modal and Rayleigh analysis. It is found that variation of fundamental periods calculated from modal analysis and Rayleigh method are quite similar.
- vii) This study indicates that there is very poor correlation between fundamental periods of three dimensional buildings with any of the parameters used to define the setback irregularity by the previous researchers or design codes.

### **5.3 SCOPE OF FUTURE STUDY**

- i) This study could not conclude on the appropriate parameter defining the irregularity in three-dimensional multi-storeyed setback buildings. There is a scope to investigate different parameters either geometrical or structural or combination of both to define the setback irregularity.

- ii) The present study is limited to reinforced concrete (RC) multi-storeyed building frames with setbacks only in one direction. There is a future scope of study on three dimensional building models having setbacks in both of the horizontal orthogonal directions.

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## **LIST OF PAPERS SUBMITTED ON THE BASIS OF THIS THESIS**

### **PRESENTATION IN CONFERENCES**

1. Vinay Mohan Agrawal and Pradip Sarkar, “Effect of Irregularity on the fundamental period of setback buildings”, 11<sup>th</sup> International conference, RASD 2013 Pisa, Italy (Accepted).